

Modeling and Simulation of Non-Isothermal Gas-Assisted Injection Molding for Non-Newtonian Fluids

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By

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Abstract

Gas-assisted injection molding (GAIM) is a process where a molten polymer is injected into a cooled mold cavity followed by an injection of gas, allowing the polymer to fill the cavity and form a hollow part as it cools and solidifies. Current GAIM simulations for non-isothermal Newtonian fluids can accurately predict the coating thickness of the polymer melt. However, no existing models predict the behavior of non-isothermal non-Newtonian fluids. The proposed simulations use known material and system properties and process variables such as mold temperature, melt temperature, and pressure, to calculate the temperature and velocity profiles to predict the coating thickness using derived relations. Experiments have been performed to determine the effect of temperature gradients and rheological behavior of the penetrating fluid. The experimental data was compared against the simulation for both Newtonian and shear-thinning fluids utilizing various viscosity models to describe the behavior of the fluids and showed that the simulations were reliable in predicting the fractional coverages at higher rowlengths. Velocity and temperature profiles were calculated in which the non-isothermal behavior of the system was shown to be predicted by the simulation. In addition, viscosity vs shear rate plots were created to validate the assumed behavior of the tested fluids for more accurate modeling. Simulated vs experimental results are plotted for the cross-exponential viscosity model for both polycarbonate and polystyrene at various rowlengths, and additionally the Ellis and power law viscosity models for polystyrene. An accurate simulation will help to reduce production costs and improve product quality.

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1 Introduction

Gas-assisted injection molding is a process involving the penetration of an inviscid gas through a viscous, non-Newtonian polymer under non-isothermal conditions. The process begins with the injection of a polymer melt into a hollow mold. After a certain amount of time, termed the delay time, a gas is inserted into the mold. This injection produces a bubble that forces the polymer melt against the wall. Once the polymer melt has cooled and solidified, the result is a hollow part that is light and relatively cheap to make compared to other conventional methods. An image of this process can be seen in Figure 1.

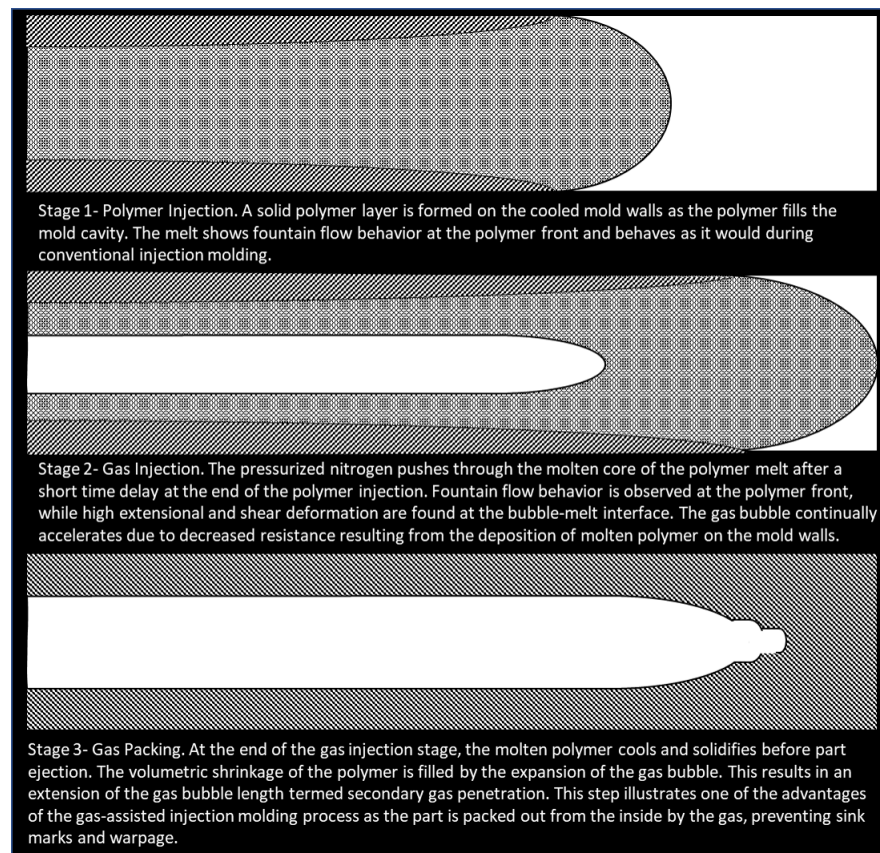


Figure 1: The Three Stages of the Gas-Assisted Injection Molding Process.

GAIM was developed to overcome the limitations of conventional injection molding. The GAIM process is able to produce plastic parts that are up to 30% lighter, have higher strength to weight ratios, are stiffer, and show ascetic improvements when compared to

conventional injection molded parts. Since the process consists of an incomplete injection of molten polymer, or short shot, followed by an injection of compressed nitrogen gas, a hollow part that is up to 50% gas, by volume, is produced. Parts molded by conventional injection molding technology are packed out through high screw holding pressures, but can only be packed if the gates to the mold cavity are not frozen. A gas injection molded part, however, is packed from within by the inert, high pressure gas throughout the cooling cycle producing better dimensional stability.

GAIM provides further benefits as well. The gas-assist technique is ideal for adding thick, hollowed-out sections to otherwise thin-walled parts and creating these thin-walled parts with low clamp tonnage, reducing both tooling cost and required injection molding machine size.[7] These advantages allow for functional, economical, and manufacturing benefits. Parts made by GAIM are commonly used in the plastics processing, and medical device industries.

Simulating the behavior of GAIM processes is necessary when attempting to control mold design and obtain a good quality product. GAIM simulation software is available but all these softwares are based on 3D models. The detriments to these models are that full-scale 3D modeling requires discretization of thin parts resulting in unreasonable computation times and high calculation instabilities. Previous work has been done in which alternative simulation models were created to predict coating thickness of non-Newtonian fluids in a free convection setting.

This work expanded upon the model for non-Newtonian fluids through the incorporation of the Cross-Exponential, Ellis, and Power Law models. For the Ellis and Cross-Exponential models, the modeling is based on the use of Mathematica to solve non-linear ordinary differential equations, as well as the use of dimensionless variables. The Power Law model utilizes FDM in its solution. In addition, this work compared the simulated results with experimental data based on varying process conditions in order to analyze the accuracy of the model at each set of conditions. The simulation results are compared with experimental data collected by Kaminski[9] for verification.

2 Literature Review

2.1 Newtonian Isothermal Gas-Liquid Modeling Experiments

Previous work has been done in which the behavior of GAIM processes under isothermal systems have been studied. Fairbrother conducted the first experiments in this area with viscous Newtonian fluids[5]. His studies determined a relationship between capillary number (Ca) and fractional coverage (m). Fractional coverage is the amount of fluid left behind on the mold after the gas bubble has penetrated through. Fairbrother was able to develop an equation relating fractional coverage and capillary number. This relation can be seen in Equation 1.

$$m = Ca^{0.5} \quad (1)$$

This equation holds true for capillary numbers up to 0.009. An equation for fractional coverage in tube shaped geometries can be seen in Equation 2.

$$m = \frac{A_p}{A_t} = 1 - \frac{R_x^2}{R_t^2} \quad (2)$$

In this equation, A_p is the cross-sectional area of the polymer deposited on the tube walls by the bubble, A_t is the cross-sectional area of the entire tube, R_x is the radius of the gas bubble, and R_t is the radius of the tube. Capillary number is the ratio of viscous to surface tension forces acting across an interface between a liquid and a gas. A relation for capillary number can be found in Equation 3.

$$Ca = \frac{\eta U_b}{\sigma} \quad (3)$$

where η represents the viscosity of the fluid, U_b is the velocity of the bubble, and σ is the fluid surface tension.

Taylor went on to develop this investigation to extend the range of capillary numbers up to 2.0[14]. The next year, Cox went on to extend the research to higher capillary numbers and determined that fractional coverage reached an asymptotic value of $m = 0.6$ for Newtonian fluids with capillary numbers greater than 10[4].

2.2 Non-Newtonian Isothermal Gas-Liquid Modeling Experiments

Poslinski and Coyle went on to conduct similar isothermal experiments using silicone liquid pastes. These pastes exhibited Bingham behavior, in which the fluids displayed shear-thinning behavior at low shear rates and a Newtonian plateau at higher shear rates[11][12]. Poslinski and Coyle used an isothermal computer simulation to model the penetration of a constant velocity gas bubble through an isothermal, shear-thinning fluid[12]. Their results showed good correlations with their experimental results. Huyzak and Koelling went on to study the same system with Boger fluids[8]. The Boger fluids showed fractional coverages that matched the Newtonian fluid relations at low capillary numbers, but increased due to higher extension rates at the bubble tip above a capillary number of six. The fractional coverage ultimately asymptoted to 0.75 around $Ca \approx 200$. The higher fractional coverages were determined to be due to the elastic properties of the Boger fluids, while shear-thinning behavior produced lower asymptotic isothermal fractional coverages. Gauri and Koelling extended the study of long gas bubbles in viscoelastic fluids in capillary tubes[6]. Kaminski went on to thoroughly study the effects of multiple process parameters on the GAIM process [9]. Kaminski also developed numerical simulations of GAIM assuming isothermal conditions.

Yijie Wang then went to to develop models for Newtonian fluids exhibiting non-Newtonian behavior with a varying temperature gradient at high capillary numbers[15]. Chen improved numerical computational times for existing simulation methods[2]. Shen studied 2D gas channel systems with penetrating gas bubbles[13]. Belblidia established correlations for the prediction of fractional coverages in a straight 2D cylindrical tube[1]. Li produced a surface model regarding the simulation of the filling process in GAIM[10]. A significant amount of experimentation and simulation has been completed for the GAIM process but simulations of non-Newtonian fluids under non-isothermal conditions at high capillary numbers have hardly been studied.

3 Methodology

3.1 Experimental Apparatus

The experimental work utilized a spiral mold cavity of 12.7 mm diameter with a flowlength of 585mm for two different transparent, amorphous, injection molding compounds: polystyrene (PS), and polycarbonate (PC). The properties of these materials will be discussed in Section 3.2. An image of the spiral mold cavity can be found in Figure 2. The mold itself was comprised of steel with embedded cooling channels that ran parallel to the mold cavity.

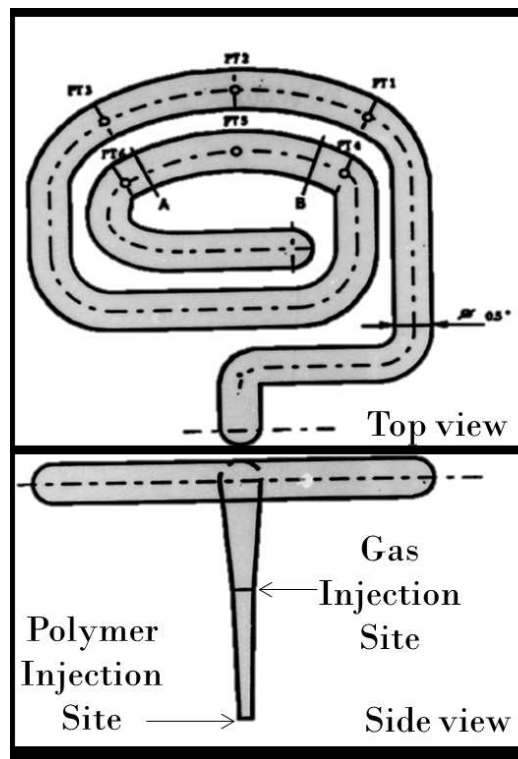


Figure 2: Spiral Mold Geometry Used in Gas-Assisted Injection Molding Trials.

Kaminski conducted experiments through a single factor matrix, in which each variable was changed to high and low settings, while keeping others constant. Table 1 lists the parameters varied at low, high, and base settings for the experiments. Each of the collected spiral samples were cross-sectioned and measured for wall thickness at different points along the flowlength. Six pressure transducers were used in order to determine the gas bubble

Table 1: Varied Parameters in Experiments for Each Material.

	PS			PC		
Parameter	Low	Base	High	Low	Base	High
Piston Speed [$\frac{in}{sec}$]	1.8	3.1	10.0	1.8	3.1	10.0
Delay Time [sec]	0.62	2.37	4.37	1.6	2.4	4.0
Pre-Charge Gas Pressure [psi]	200	350	500	175	325	500
Shot Size [inch]	2.85	3.0	3.15	2.9	3.0	3.1
Ram Speed [$\frac{in}{sec}$]	0.86	1.14	2.0	0.75	1.0	1.5
Mold Temperature [deg F]	95.6	115.6	136.6	-	180	-
Melt Temperature [deg F]	471.5	485.6	511.7	608.7	623.6	637.6

velocities and cooling times in the closed mold cavity and mounted along flush in the full spiral mold cavity at various locations. These locations can be seen in Figure 2. Video recording was also used to capture and analyze the movement of the gas and polymer fronts in the closed mold cavity.

3.2 Fluid Characterization

Two transparent injection molding grade compounds were studied: a general purpose polystyrene, Dow Styron 685 D (PS); a high viscosity polycarbonate, GE Lexan 101 (PC). Each compound exhibited unique rheological properties which were exploited to study the effects of their properties on the resulting fractional coverage of the molded test spirals. Table 2 lists various thermal properties of each material.

The fluids were characterized using the Cross-Exponential model. The shear rate dependent viscosity of each material at the base melt temperature can be seen in Figure 3. The Cross-exponential viscosity model is defined by Equation 4.

$$\eta = \frac{\eta_o}{1 + \left(\frac{\eta_o \dot{\gamma}}{\tau^*}\right)^{1-n}} \quad (4)$$

Table 2: Thermal Properties of the Investigated Polymers.

Polymer	Polystyrene	Polycarbonate
Density [$\frac{kg}{m^3}$]	940	1017
Heat capacity [$\frac{J}{kgK}$]	2100	2052
Thermal conductivity [$\frac{W}{mK}$]	0.150	0.320
Thermal diffusivity [$\frac{m^2}{s}$]	7.5×10^{-8}	1.53×10^{-7}

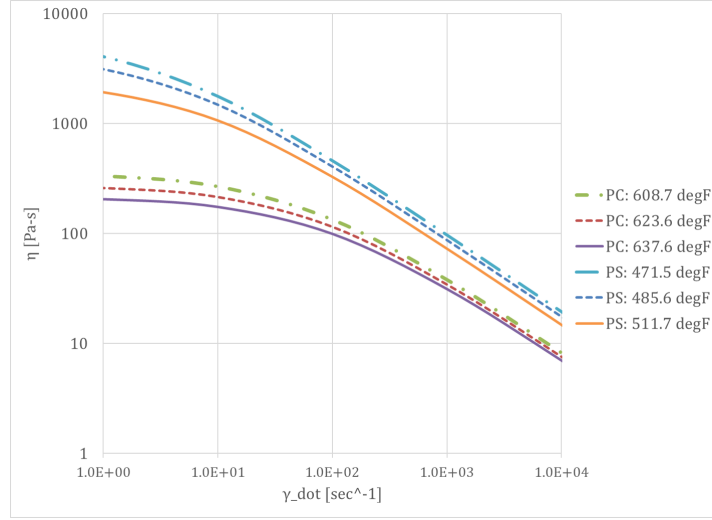


Figure 3: Shear Rate Dependent Viscosities of the Studied Polymers at the Tested Injection Temperatures.

where

$$\eta_o = B \exp\left(\frac{T_b}{T}\right), \quad (5)$$

η is the viscosity, T is the temperature, η_o is the zero shear-rate viscosity, $\dot{\gamma}$ is the shear rate, T_b is the temperature sensitivity coefficient, τ^* is the shear-stress level in relation to the transition between the Newtonian and shear-thinning regimes, and n is the shear thinning coefficient. It should be noted that the $1-n$ term in this equation is equal to the power law index. This relation will be utilized in the determination of asymptotic isothermal fractional coverage for non-Newtonian fluids. Table 3 lists values for the parameters of the Cross-Exponential model for both fluids.

Experimental work determined the tested shear rates to range from 20 s^{-1} to 4000 s^{-1} .

Table 3: Cross-Exponential Parameters for All Tested Fluids.

	Polystyrene	Polycarbonate
n	0.2968	0.03727
τ^* [Pa]	1.738×10^4	9.4221×10^5
B [Pa-s]	1.6783×10^{-6}	1.0242×10^{-4}
T_b [K]	1.1376×10^4	9.0679×10^3

Coupling this fact with Figure 3 can lead to the conclusion that PS demonstrated shear-thinning behavior throughout the entirety of the experimentation while PC has an upper Newtonian plateau that extended over a majority of the range, with the transition to shear-thinning behavior beginning around 600 s^{-1} . Therefore when modeling, PC was treated as a Newtonian fluid (this is additionally supported by the fact that the power law index found for PC was ≈ 1) and PS, non-Newtonian.

3.3 Modeling of Newtonian Fluids

While GAIM offers many advantages to conventional injection molding, there are many complexities in the system that must be taken into consideration. This section will focus on the modeling of Newtonian fluids through the calculation of the temperature profile, velocity profile, and fractional coverage. An image of expected temperature and velocity profiles for non-isothermal systems can be seen in Figure 4.

3.3.1 Calculation of Temperature Profile

A non-isothermal system leads to a temperature profile similar to the one seen in Figure 4. In a non-isothermal system, the temperature profile is greatest in the middle of the tube and lowest at the wall. The system is divided into three regions where heat transfer can occur: outside the tube, inside the tube, and inside the polymer melt. In relation to the experimental apparatus, "inside the polymer melt" would refer to inside the spiral mold cavity, "inside the tube" would refer to the wall of the mold cavity, and "outside the tube"

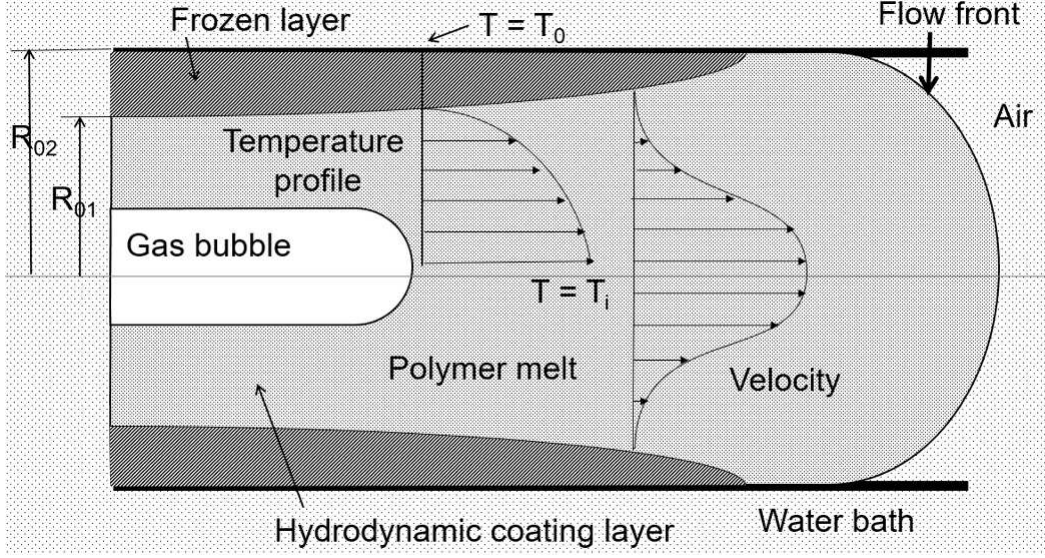


Figure 4: Bubble Penetration Through a Capillary Tube Filled with Polymer Melt Under Non-Isothermal Conditions.

would refer to the remaining portion of the steel mold.

The heat transfer outside the tube is a continuous process across the surrounding environment. This system is simplified by assuming the heat transfer occurs simply along the immediate surrounding area of the tube. Based off this assumption, the governing equation for heat transfer in this region is:

$$-k_s \frac{\partial T_s}{\partial r} \Big|_{r=R_{out}} = h(T_w - T_B) \quad (6)$$

where k_s is the thermal conductivity of stainless steel, T_s is the temperature of the steel-water interface, R_{out} is the radius of the outer tube, h is the convective heat transfer coefficient of the assumed thin layer coating the tube, T_w is the temperature of the wall, and T_B is the temperature of the bulk fluid.

Finite difference method is then used to determine the temperature of the steel/water interface.

$$T_{s(N,j+1)} = T_{s(N,j)} + \frac{2\alpha_s dt}{dr_2^2} * \left[T_{s(N,j+1)} - T_{s(N,j)} - \left(\frac{R_{out} + \frac{dr_2}{2}}{R_{out}} \right) \left(\frac{h dr_2}{k_s} \right) (T_{s(N,j)} - T_B) \right] \quad (7)$$

where $T_{s(N,j+1)}$ is the temperature of the steel/water interface at time step j , dt is the specified

time step, dr_2 is the specified radial step change, and α_s is the thermal diffusivity of the steel.

The main type of heat transfer taken into consideration was forced convection due to the setup of the experimental apparatus. Due to this assumption, the convective heat transfer coefficient is determined by Equation 8[3].

$$\bar{Nu} = A * \frac{Re^{1/2} Pr^{1/3}}{\left[1 + \left(\frac{0.4}{Pr}\right)^{2/3}\right]^{1/4}} = \frac{2R_{out}\bar{h}}{k_f} \quad (8)$$

where \bar{Nu} is the average Nusselt number, Re is the Reynolds number, Pr is the Prandtl number, A is a constant equal to 0.62, and k_f is the thermal conductivity of the bulk fluid. At this point in the simulation, velocity is not yet known making the Reynolds number calculation difficult. The influence of Reynold's number on the system was studied and found to have little to no impact on the temperature profile. Therefore, a representative value for Reynold's number was chosen for the simulation.

The heat transfer inside the tube is then taken into consideration. The governing equation for this region is

$$\frac{\partial T_s}{\partial t} = \alpha_s \left(\frac{\partial^2 T_s}{\partial r^2} + \frac{1}{r} \frac{\partial T_s}{\partial r} \right). \quad (9)$$

Again, through the implementation for FEM, Equation 9 becomes

$$T_{s(i,j+1)} = T_{s(i,j)} + \frac{\alpha_s dt}{r_{s(i)} dr_2^2} * \left[\left(\frac{r_{s(i)} + r_{s(i-1)}}{2} \right) T_{s(i-1,j)} - 2r_{s(i)} T_{s(i,j)} + \left(\frac{r_{s(i+1)} + r_{s(i)}}{2} \right) T_{s(i+1,j)} \right] \quad (10)$$

where $T_{s(i,j)}$ is the temperature of the steel at node i at any time step j and $r_s(i)$ is the radial position in the steel at node i .

The following initial and boundary conditions were applied:

$$IC1 : @t = 0, T_{s(i,0)} = T_i \quad (11)$$

where T_i is the initial temperature of polymer melt.

$$BC1 : @r = R_{out}, T_{s(N,j)} = T_c \quad (12)$$

where T_c is the initial temperature of the cooling fluid.

$$BC2 : @r = R_{in}, q_{cond(n-1 \rightarrow n)} A_1 + q_{cond(n+1 \rightarrow n)} A_2 = q_{stored(n)} A_3 \quad (13)$$

FEM is applied on the last boundary condition to obtain

$$\begin{aligned} T_{s(i,j+1)} = T_{s(i,j)} &+ \frac{2k_s dt (R_{in} + \frac{dr_2}{2}) (T_{s(i+1,j)} - T_{s(i,j)})}{dr_2 [(R_{in} + \frac{dr_2}{4}) \rho_s C_{p,s} dr_2 + (R_{in} - \frac{dr_1}{4}) \rho_s C_{p,p} dr_1]} \\ &+ \frac{2k_p dt (R_{in} - \frac{dr_1}{2}) (T_{s(N-1,j)} - T_{s(i,j)})}{dr_1 [(R_{in} + \frac{dr_2}{4}) \rho_s C_{p,s} dr_2 + (R_{in} - \frac{dr_1}{4}) \rho_s C_{p,p} dr_1]} \end{aligned} \quad (14)$$

where dr_1 is the radial step change in the polymer.

The last region in which heat transfer is considered is region inside the polymer. The governing equation for this region can be found in Equation 15.

$$\frac{\partial T_p}{\partial t} = \alpha_p \left(\frac{\partial^2 T_p}{\partial r^2} + \frac{1}{r} \frac{\partial T_p}{\partial r} \right) \quad (15)$$

where α_p is the thermal diffusivity of the polymer, and T_p is the temperature of the polymer.

With the application of FEM, Equation 15 becomes

$$T_{p(i,j+1)} = T_{p(i,j)} + \frac{\alpha_p dt}{r_{p(i)} dr_1^2} * \left[\left(\frac{r_{p(i)} + r_{p(i-1)}}{2} \right) T_{p(i-1,j)} - 2r_{p(i)} T_{p(i,j)} + \left(\frac{r_{p(i+1)} + r_{p(i)}}{2} \right) T_{p(i+1,j)} \right] \quad (16)$$

where $r_{p(i)}$ is the radial position in the polymer at node i.

The applied initial and boundary conditions in this region were:

$$IC2 : @t = 0, T_{p(i,0)} = T_i \quad (17)$$

$$BC3 : @r = 0, -k_p \frac{\partial T_p}{\partial r} \Big|_{r=0} = 0 \quad (18)$$

Applying FEM to the last boundary condition, Equation 18 becomes

$$T_{p(i,j+1)} = T_{p(i,j)} + 4 \left(\frac{\alpha_p dt}{dr_1^2} \right) (T_{p(i+1,j)} - T_{p(i,j)}). \quad (19)$$

The implementation of these equations presumes that the calculated temperature profile is that of the region far in front of the penetrating bubble. In order to determine the temperature profile of the polymer near the bubble, it is assumed that as the bubble passes

through the polymer at high velocities, the fluid directly in front of the bubble is relocated between the region between the bubble and inner wall. Additionally, the region upon which the temperature profiles are changing from static flow to flow in front of the bubble are neglected for simplification. We assume the temperature profile in front of the bubble tip and far in front of the bubble are the same. Through the implementation of these assumptions and by calculating the radial position of the bubble curve at each node, the squeezed temperature profile is calculated. Temperature profiles for each time step in the system are determined as well.

3.3.2 Calculation of Velocity Profile

From Figure 4, it can be seen that the continuous temperature change in the system causes the velocity profile to deviate from the common parabolic shape expected for laminar flow in a pipe for Newtonian fluids. To begin the calculation of the velocity profile, pseudo-steady state is assumed. The momentum balance for flow in a tube was used to solve for the velocity profile.

$$\frac{\partial}{\partial r} \left(r\eta \frac{\partial u_z}{\partial r} \right) = \frac{\Delta P r}{L} \quad (20)$$

where ΔP is the pressure gradient and L is the tube length. From Equation 20, it can be observed that this equation has no temperature dependence. To account for the temperature dependence in our system, Equation 5 is coupled with the momentum balance. Recall that the zero shear viscosity is the viscosity measured for shear deformation at low shear rates. This fact coupled with the fact that Newtonian behavior was observed for all materials at low shear rates supports the coupling of Equation 5 with Equation 20. Therefore, Equation 20 becomes

$$\frac{\partial}{\partial r} \left(rB * \exp\left(\frac{T_b}{T}\right) \frac{\partial u_z}{\partial r} \right) = \frac{\Delta P r}{L}. \quad (21)$$

Applying FEM on Equation 21 leads to

$$u_{z(i,j)} = u_{z(i-1,j)} + \frac{1}{2(N-1)} \left(\frac{\Delta P r_{p(i)}}{2LB} \right) * \left[\frac{r_{p(i)}}{\exp(\frac{T_b}{T_{p(i,j)}})} + \frac{r_{p(i-1,j)}}{\exp(\frac{T_b}{T_{p(i,j)}})} \right]. \quad (22)$$

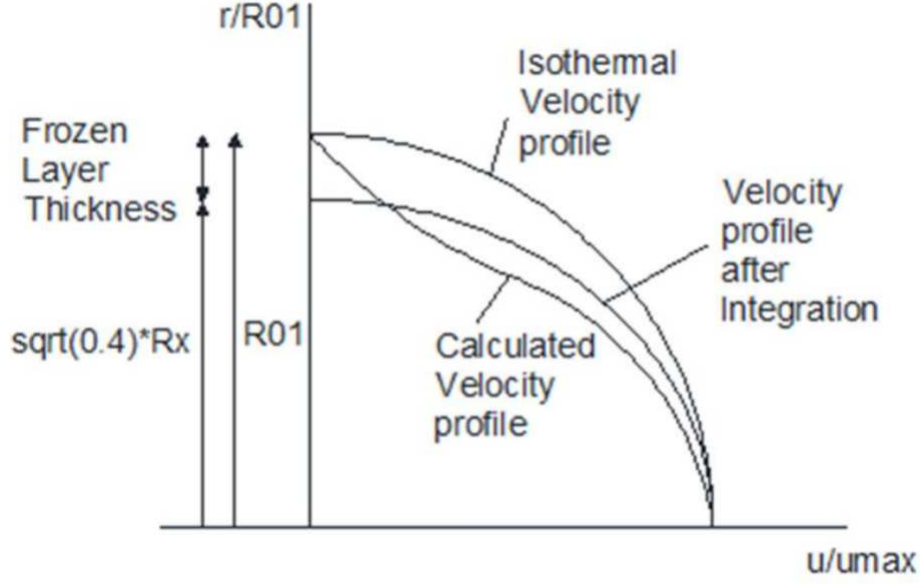


Figure 5: Isothermal, Calculated, and Assumed Velocity Profiles.[9]

The following boundary conditions were applied:

$$BC4 : u_z|_{r=R_{in}} = 0 \quad (23)$$

$$BC5 : \left. \frac{du_z}{dr} \right|_{r=0} = 0 \quad (24)$$

3.3.3 Calculation of Fractional Coverage

To begin the calculation of the fractional coverage, one major assumption is made. This assumption is best depicted pictorially and can be seen in Figure 5. The velocity profiles of non-isothermal systems are not parabolic. This provides many complexities when modeling. In order to simplify our calculations, we assume the non-isothermal system is equal to an isothermal system with a frozen layer. The now parabolic shaped velocity profile can be describe by the equation:

$$u^* = 1 - \left(\frac{r}{R_x} \right)^2 \quad (25)$$

where u^* is the parabolic velocity profile and R_x is the radius of the bubble.

The volumetric flow rates for both the nonisothermal system and the isothermal system

with the frozen layer are assumed to be equal. This can be described by Equation 26.

$$2\pi \int_0^{R_{in}} ru_z^* dr = 2\pi \int_0^{R_x} ru^* dr \quad (26)$$

This equation can be integrated and solved to determine R_x . Once R_x is calculated, fractional coverage can be determined with the following equation:

$$m = 1 - (1 - m_\infty) \left(\frac{R_x}{R_{in}} \right)^2 \quad (27)$$

where m_∞ is the asymptotic isothermal fractional coverage. For Newtonian fluids at high capillary numbers, this value has been found to be 0.6[4]. This method was implemented for the determination of fractional coverage for polycarbonate. The written MATLAB code can be found in Appendix B. The results from this implementation are discussed in the next chapter.

3.4 Modeling of Non-Newtonian Fluids

When determining the temperature for the system with non-Newtonian fluids, the calculations provided in the previous section are still valid. Variations arise when determining a proper model for the viscosity of the fluid. When modeling non-Newtonian fluids, three viscosity models were applied: the cross-exponential model, the Ellis model, and the power law model. Their implementations are described in this section.

3.4.1 Use of Cross-Exponential Model

For non-Newtonian fluids, the assumption of Equation 5 to model the viscosity is no longer valid due to the shear-thinning behavior that occurs at higher shear rates. Therefore, Equation 4 was used. Combining Equations 4 and 20 gives us

$$\frac{\partial}{\partial r} \left(r \frac{\eta_o}{1 + \left(\frac{\eta_o \dot{\gamma}}{\tau^*} \right)^{1-n}} \dot{\gamma} \right) = \frac{\Delta P r}{L}. \quad (28)$$

To employ FEM on this equation, $\dot{\gamma}$ must be solved for explicitly. However, due to the nonlinearity of this equation, no analytical solution can be determined. Therefore, this equation was solved numerically.

When solving this equation, it is convenient to introduce the following dimensionless variables:

$$v^* = \frac{\Delta P R_{in}^2}{L \eta_o} \quad (29)$$

$$r^* = \frac{r}{R_{in}} \quad (30)$$

$$E^* = \frac{\eta_o v^*}{\tau^* R_{in}} = \frac{\Delta P R_{in}^2}{\tau^* L} \quad (31)$$

$$\phi = \frac{u_z}{v^*} \quad (32)$$

$$\psi = \frac{d\phi}{dr^*} \quad (33)$$

Therefore, the Cross-exponential expression for the viscosity momentum balance becomes

$$\frac{d}{dr^*} \{ r^* [1 + (E^* \psi)^{n-1}] \psi \} = r^*. \quad (34)$$

The boundary conditions of no-slip and symmetry about the pipe center line become as follows:

$$BC4 : \phi(1) = 0 \quad (35)$$

$$BC5 : \psi(0) = 0 \quad (36)$$

Although previous calculations had been completed in MATLAB, due to the nonlinearity of this ODE, Mathematica was used to solve this equation since most MATLAB functions are limited to solving linear ODEs. The code leading to the solution of this equation can be found in Appendix H.

It should be noted that when the ODE was made dimensionless, the only temperature dependent variable is ψ . This means that this variable is dependent on both temperature and radius. When redimensionalizing the variables, this dual dependence can be removed. The velocity profiles obtained from these calculations can be found in the next chapter.

To solve for the fractional coverage, again the assumption of equality of both a nonisothermal and isothermal system with a frozen layer is applied. When equating the volumetric flow rates, again it was found that equating the flowrates simplified the calculations. This

relation was found to be

$$\frac{Q_1^* L_1 \eta_{o, isothermal}}{R_{01}^4 \Delta P_1} = \frac{Q_x^* L_x \eta_{o, non-isothermal}}{R_x^4 \Delta P_x}. \quad (37)$$

where the subscript 1 refers to the non-isothermal system and x refers to the isothermal system. In this equation, it was assumed that the dimensionless flowrates and pressure gradients were equal. This simplified the equation and allowed for an analytical solution of R_x .

With a solution for R_x , fractional coverage was able to be solved with Equation 27. It should be noted that since the fluids considered are now non-Newtonian, the value for the asymptotic isothermal fractional coverage is not 0.6.

Poslinski and Coyle determined a relationship for fractional coverage based on the power law index[11]. This relationship was found to be

$$m_\infty = \begin{cases} -0.1516 \ln(n) + 0.7259 & n \leq 0.3 \\ -0.1428n^2 + 0.2626n + 0.4782 & n > 0.3 \end{cases} \quad (38)$$

Fractional coverage was now able to be determined and results can be found in the next chapter. The code implemented for this solution can be found in Appendix D.

Since the cross-exponential model is significant only at low shear rates, the Ellis model was implemented for a more appropriate consideration of the shear-thinning behavior of non-Newtonian fluids.

3.4.2 Use of Ellis Model

The Ellis model can be expressed with the following equation

$$\eta = \frac{\eta_o}{1 + \left(\frac{\tau_{rz}}{\tau_{1/2}}\right)^{\alpha-1}} \quad (39)$$

where α is a measure of the degree of shear thinning behavior, $\tau_{1/2}$ is the value of shear stress at which the apparent viscosity has dropped to half its zero shear value, and τ_{rz} is the shear stress. The dependence of τ_{rz} in this equation allows for the consideration of the shear-thinning behavior of the modeled material.

Polystyrene was treated as a non-Newtonian fluid in these simulations. For this reason, the three Ellis model parameters were determined for this material. These parameters can be found in Table 4.

Table 4: Ellis Model Parameters for PS.

Parameter	Value
α	3.26
$\tau_{1/2} [Pa - s^2]$	16618.6
$\eta_o [Pa-s]$	$1 \times 10^{16} e^{-0.056T[K]}$

Due to the shear-thinning behavior of non-Newtonian fluids, τ_{rz} cannot be assumed to be constant. However, since laminar flow is assumed, the shear stress profile is known (see Figure 6). From this profile, an equation for τ_{rz} can be determined if shear stress at the wall

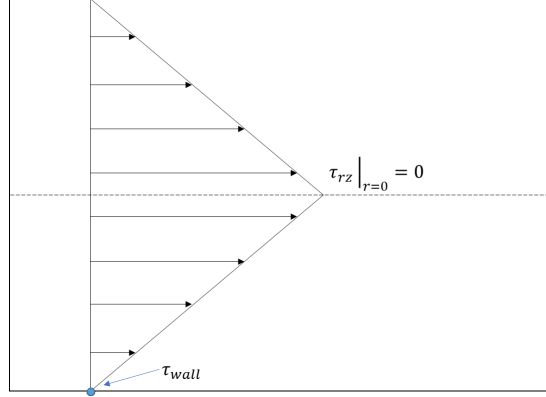


Figure 6: Shear Stress Profile for Laminar Flow in a Pipe.

is known. This relation can be found in Equation 40.

$$\tau_{rz} = \tau_{wall} \frac{r}{R_{01}} \quad (40)$$

With a relation for τ_{rz} now determined, the Ellis model can now be implemented into the momentum balance. Again, this ODE was made dimensionless for simplification purposes. The result is

$$\frac{d}{dr^*} \{r^* [1 + (e^* \Psi)^{\alpha-1}] \Psi\} = r^*. \quad (41)$$

where

$$e^* = \frac{\tau_{wall}}{\tau_{1/2}}, \quad (42)$$

$$\Psi = \frac{d\Phi}{dr^*}, \quad (43)$$

$$\Phi = \frac{u_z \nu_o}{2\tau_{wall}}. \quad (44)$$

Again, Mathematica was used to solve this ODE and the code reflecting this solution can be found in Appendix I. The dimensionless variables were used to solve for u_z and velocity profiles were determined.

Regarding the calculation of fractional coverage, the dimensionless flow rates were equated. This equation simplified to

$$R_x = R_{01} \sqrt{\frac{\tau_{wall,non-isothermal} \eta_{o,isothermal}}{\tau_{wall,isothermal} \eta_{o,non-isothermal}}}. \quad (45)$$

With R_x now determined, fractional coverage was calculated using Equations 27 and 38. The code implementing this solution is found in Appendix E and results can be found in the next chapter.

3.4.3 Use of Power Model

The power law model was also used to model the polystyrene. The power law model follows the form:

$$\eta = m\dot{\gamma}^{n-1} \quad (46)$$

where m and n are material dependent constants. The Power Law model was applied due to its simplicity during calculations.

With the coupling of the momentum balance and the viscosity equation, the following equation is reached:

$$\frac{\partial}{\partial r} \left(rm\dot{\gamma}^n \right) = \frac{\Delta P r}{L} \quad (47)$$

The application of FEM to this equation leads to:

$$u_{z(i,j)} = u_{z(i-1,j)} + \frac{1}{2(N-1)} \left(\frac{\Delta P r_{p(i)}}{2Lm} \right)^{\frac{1}{n}} * \left[(r_{p(i)})^{\frac{1}{n}} + (r_{p(i-1,j)})^{\frac{1}{n}} \right] \quad (48)$$

Previously stated boundary conditions were applied during the velocity profile calculations, Equation 26 was used to determine the bubble radius, and Equations 27 and 38. were used to calculate the fractional coverage.

4 Results

4.1 Simulation Results for Newtonian Fluids under Non-Isothermal Conditions

4.1.1 Temperature Profiles

Two temperature profiles are shown in this section to verify the non-isothermal behavior of the system. Figure 7 shows the calculated temperature profile for both the spiral mold cavity and the steel mold. It can be observed that very little heat transfer occurs through the steel mold due to its heat transfer properties and the difference in size between the spiral mold cavity and the steel mold.

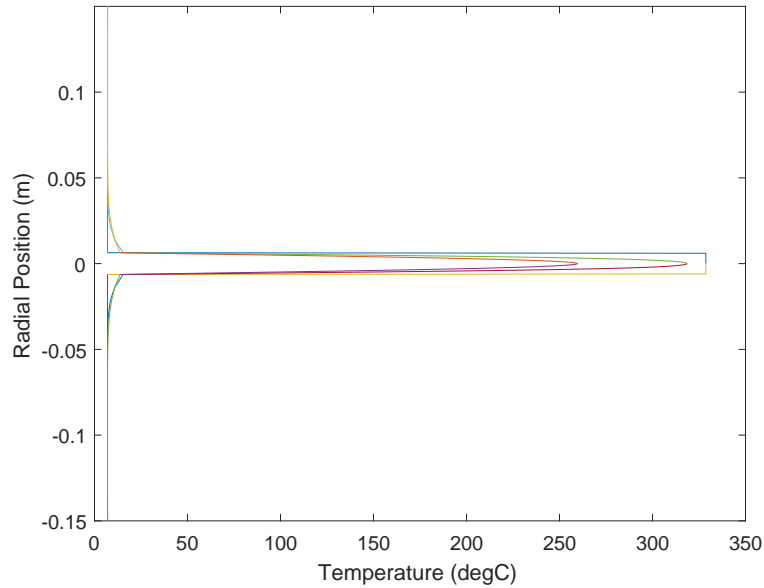


Figure 7: Temperature Profile for Both the Spiral Mold Cavity and Steel Mold for PC.

Figure 8 shows the calculated temperature profile for the spiral mold cavity. The parabolic nature of the profile as time increases confirms the non-isothermal assumption. In addition, the decreasing temperature as time increases confirms the phenomena of cooling as time increases.

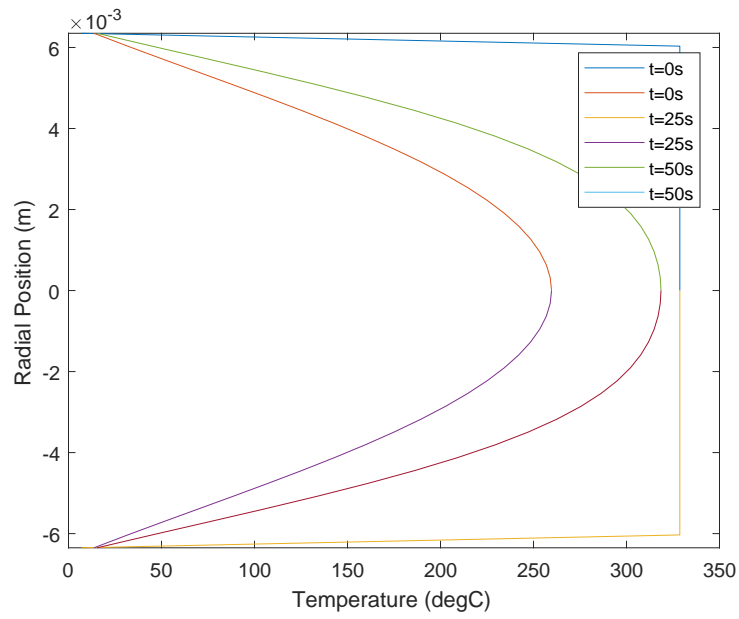


Figure 8: Temperature Profile for the Spiral Mold Cavity for PC.

4.1.2 Velocity Profiles

The normalized velocity profile was also determined for the spiral mold cavity. Figure 9 shows the calculated normalized velocity profile. Again, non-isothermal behavior is observed in the velocity profile as time goes on.

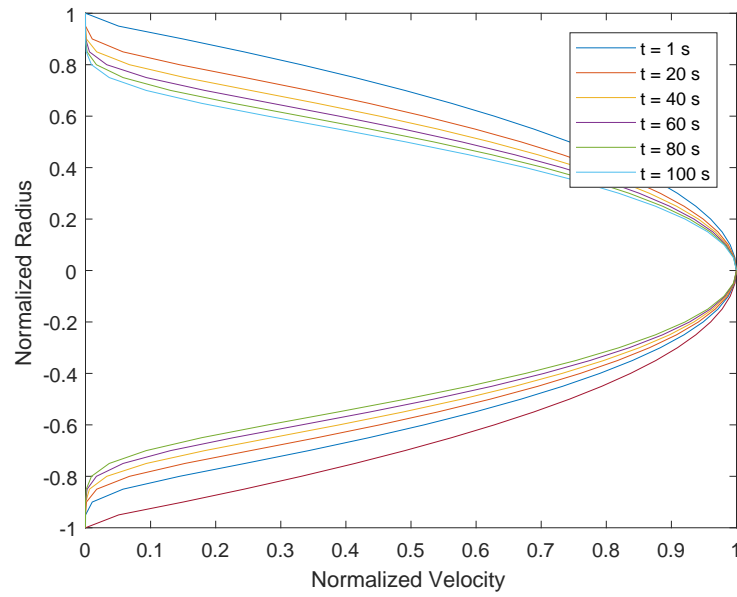


Figure 9: Normalized Velocity Profile for PC with Cross-Exponential Zero-Shear Viscosity Model.

4.1.3 \bar{m} vs Fo at Base Conditions

With the calculated temperature and velocity profiles, the fractional coverage at each point was calculated and plotted against Fourier number to observe an estimate of the simulation vs the experimental data at all experiments. This simulation data may not necessarily be true for all points due to the changing conditions with each experiment. However, plotting all the points vs the simulation at base conditions allowed for an initial analysis of points to study in the system. Figure 10 shows the determined fractional coverage at base conditions using the Cross-exponential zero-shear viscosity model vs Fourier number. From the plot, it was determined to compare the simulated vs experimental fractional coverages at 90mm, 285mm, 343mm, and 505mm.

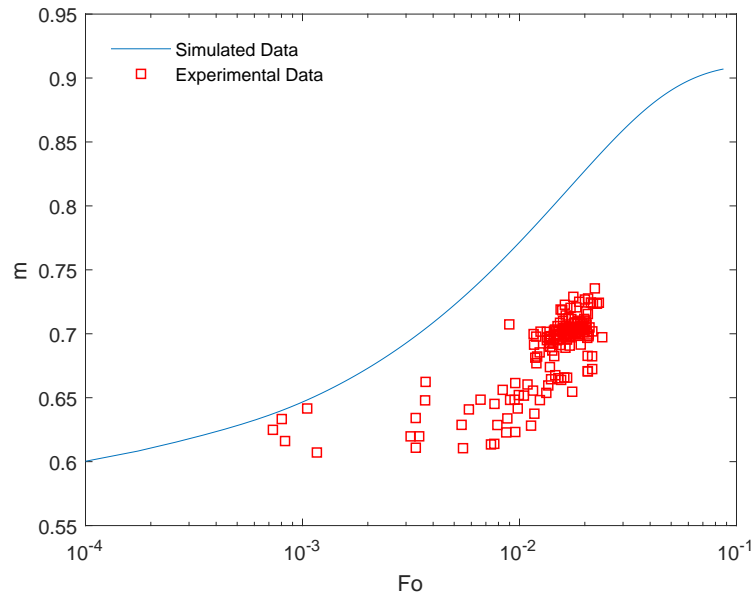


Figure 10: \bar{m} vs Fo for Polycarbonate data with Cross-Exponential Zero-Shear Viscosity Model at Base Processing Conditions.

4.2 Comparison of Simulation and Experimental Results for Fractional Coverage of Newtonian Fluids

4.2.1 Effect of Shot Size

The first varying setting examined was shot size. In order to properly determine each point, first the viscosity and shear rate were determined to confirm the Newtonian behavior assumed. Figure 11 shows the shear rate vs viscosity for each point examined. The melt temperatures were approximately the same for each point causing the viscosity lines to lie very close together. Therefore, in order to easily see the difference between each simulated point, Figure 12 shows a zoomed in version of Figure 11. From these plots, we can conclude that all the examined points lie in the Newtonian region of the curve, making our assumption of Newtonian behavior valid.

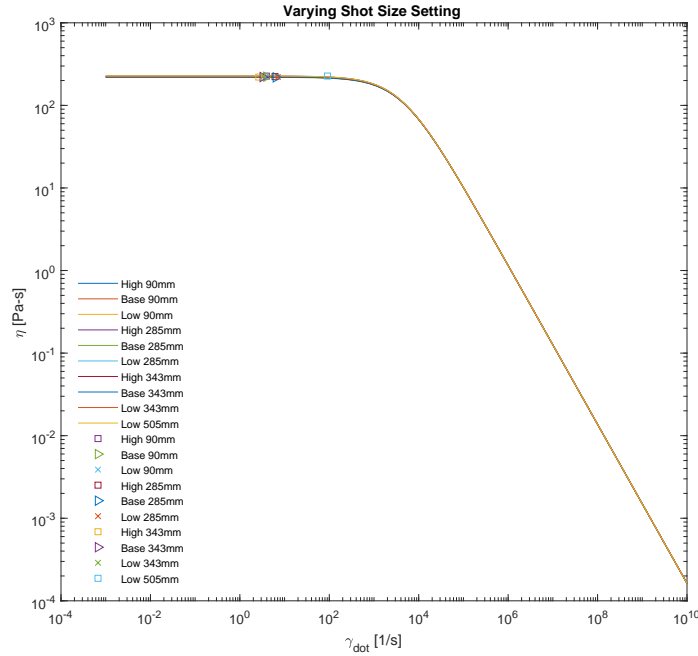


Figure 11: Viscosity vs Shear Rate for Each Simulated Point Varying Shot Size Setting for PC where High = 2.9in, Base = 3.0in, and Low = 3.1in.

With the validation of the assumption of Newtonian behavior, the fractional coverages

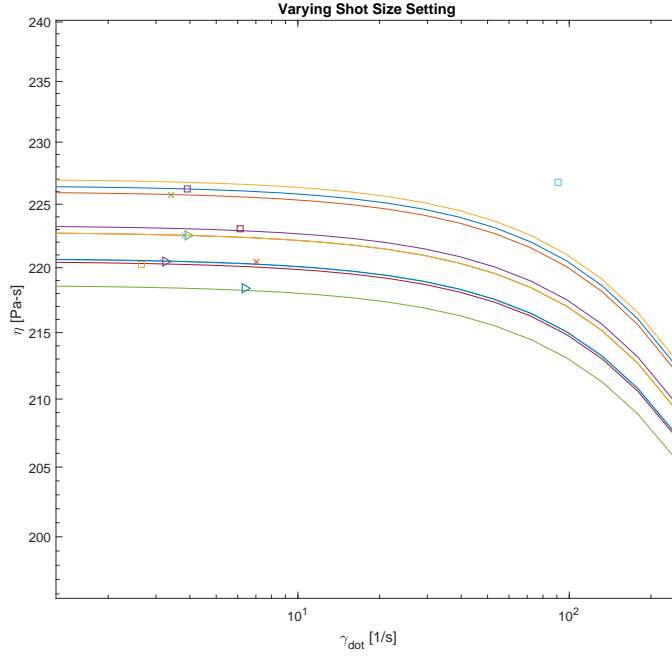


Figure 12: Zoomed Viscosity vs Shear Rate for Each Simulated Point Varying Shot Size Setting for PC where High = 2.9in, Base = 3.0in, and Low = 3.1in.

were then compared for each point as seen in Figure 13. The simulation tends to predict fractional coverages that are higher than expected. For rowlengths 90mm, 285mm, and 343mm, the simulation tends to predict fractional coverages around 0.9. This prediction is more accurate for the high shot size setting although still higher than the experimental data. For 505mm, the point seems to be closer to the perfect simulation line. This could be due to the acceleration in velocity as the bubble travels down the tube which would increase the shear rate of the fluid making it slightly more non-Newtonian. However, the data points tend to be clustered together making these trends difficult to observe. No data was available for high and base shot size settings at 505mm due to the fact that for these systems, the bubble would not reach this rowlength based on the increase in shot size.

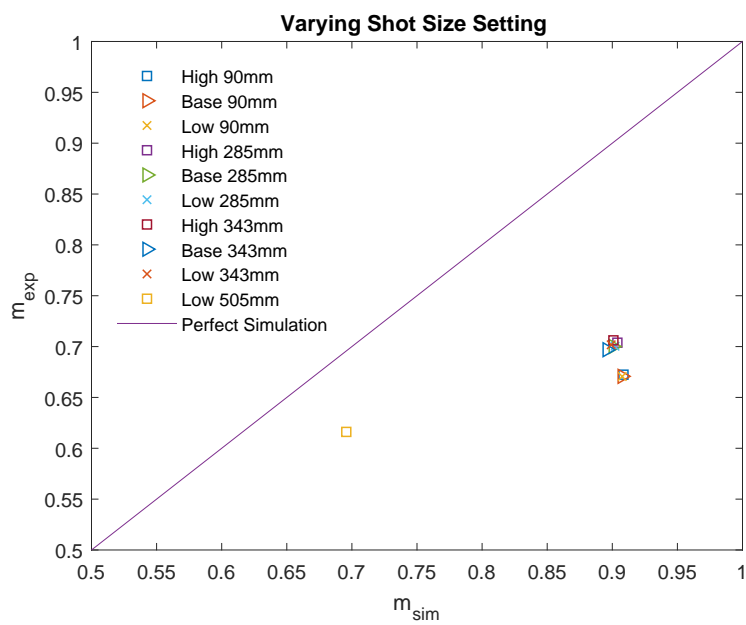


Figure 13: Simulated vs Experimental Fractional Coverage for PC with Varying Shot Size Setting where High = 2.9in, Base = 3.0in, and Low = 3.1in.

4.2.2 Effect of Delay Time

The next varied setting examined was delay time. Again shear rate vs viscosity was plotted for each simulation and Newtonian behavior was observed. Figure 14 shows the viscosity vs shear rate for each point and Figure 15 shows a zoomed in version of Figure 14.

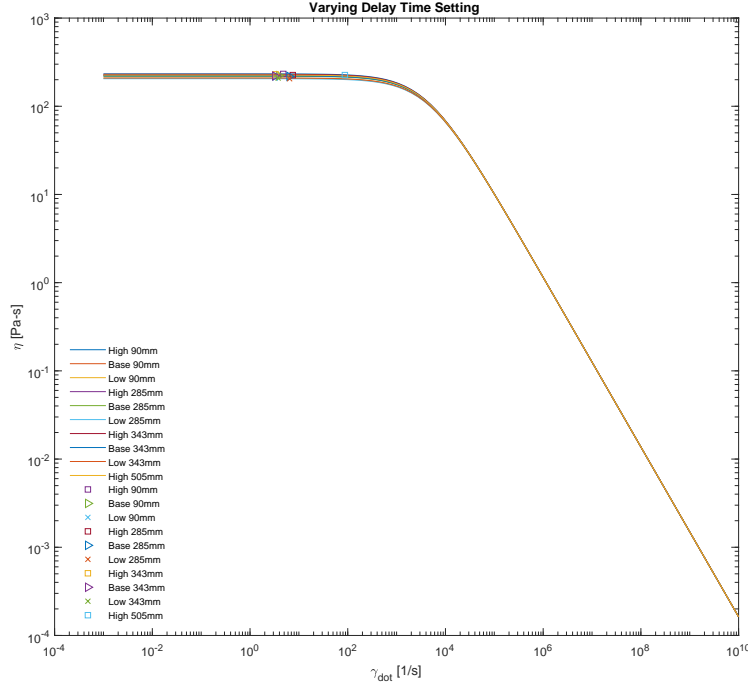


Figure 14: Viscosity vs Shear Rate for Each Simulated Point Varying Delay Time Setting for PC where High = 1.6sec, Base = 2.4sec, and High = 4.0sec.

Figure 16 shows the simulated vs experimental fractional coverage with varying delay time for multiple rowlengths. Unlike with the variation of shot size setting, these points are much less clustered, making trends slightly easier to observe. Again, the simulation tended to predict higher fractional coverages compared to the experimental data. However, we can observe that the simulation predicts more accurate fractional coverages for high delay time setting than the low setting. Also while m_{sim} at rowlengths 90mm, 285mm, and 343mm, tended to be much higher, at 505mm, the predicted fractional coverage is much closer to the experimental fractional coverage value.

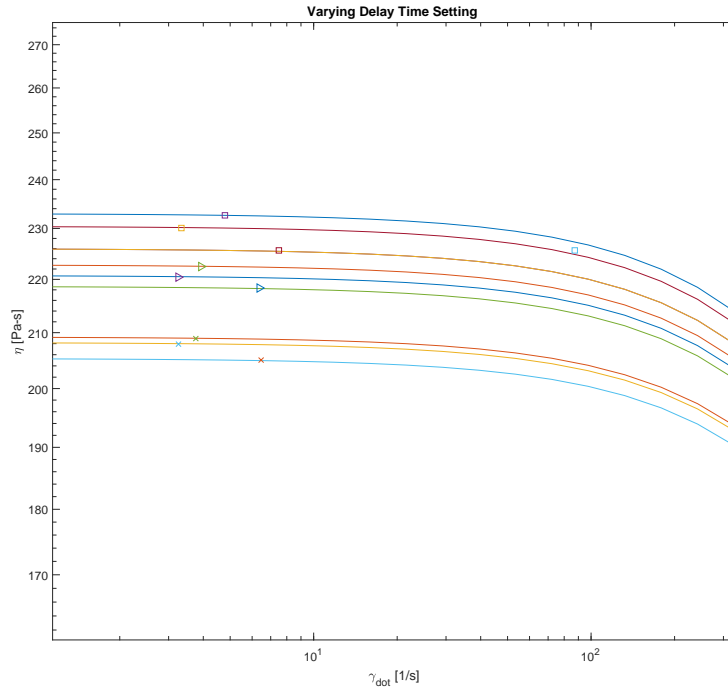


Figure 15: Zoomed Viscosity vs Shear Rate for Each Simulated Point Varying Delay Time Setting for PC where High = 1.6sec, Base = 2.4sec, and High = 4.0sec.

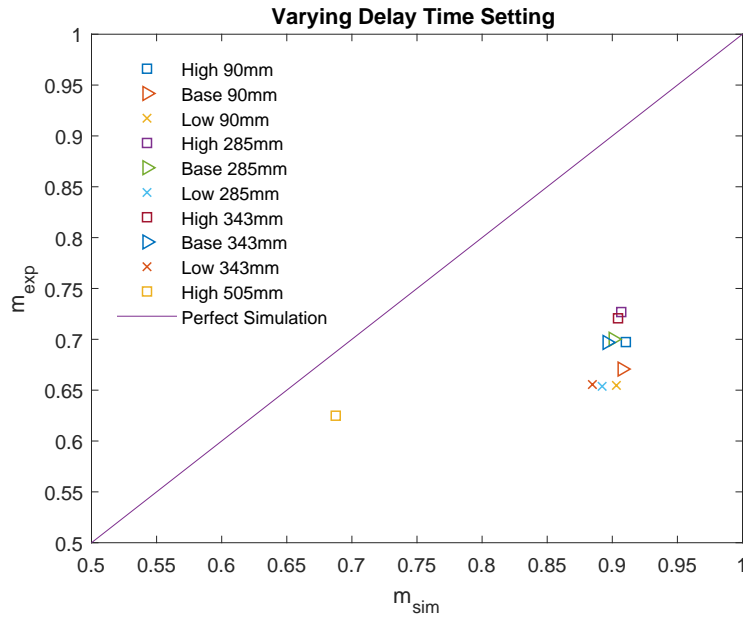


Figure 16: Simulated vs Experimental Fractional Coverage for PC with Varying Delay Time Setting where High = 1.6sec, Base = 2.4sec, and High = 4.0sec.

4.2.3 Effect of Pre-charge Gas Pressure

The effect of varying pre-charge gas pressure setting at various rowlengths was also examined. Figure 17 shows the viscosity vs shear rate for each simulated point. It is observed that all points lie in the Newtonian region of the curve. Figure 18 shows a zoomed in version of the viscosity vs shear rate plot.

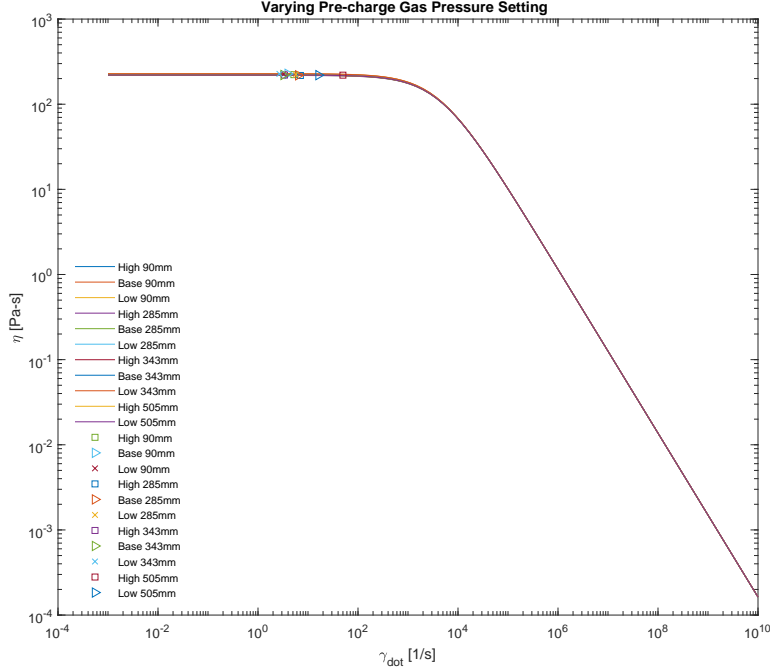


Figure 17: Viscosity vs Shear Rate for Each Simulated Point Varying Pre-charge Gas Pressure Setting for PC where High = 500psi, Base = 325psi, and Low = 175psi.

Figure 19 shows the simulated vs experimental fractional coverage as pre-charge gas pressure setting and rowlength are changed. It is observed that for the low setting, the simulated prediction tends to be slightly better due to the higher experimental fractional coverage values. Again, rowlengths 90mm, 285mm, and 343mm tend to be clustered around $m_{sim} \approx 0.9$ while at 505mm, the simulated fractional coverages tend to be closer to the experimental values.

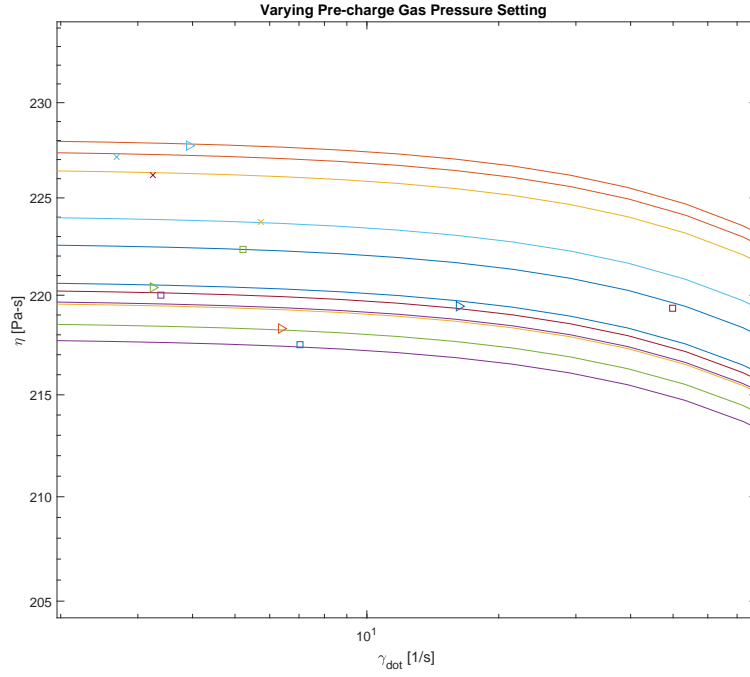


Figure 18: Zoomed Viscosity vs Shear Rate for Each Simulated Point Varying Pre-charge Gas Pressure Setting for PC where High = 500psi, Base = 325psi, and Low = 175psi.

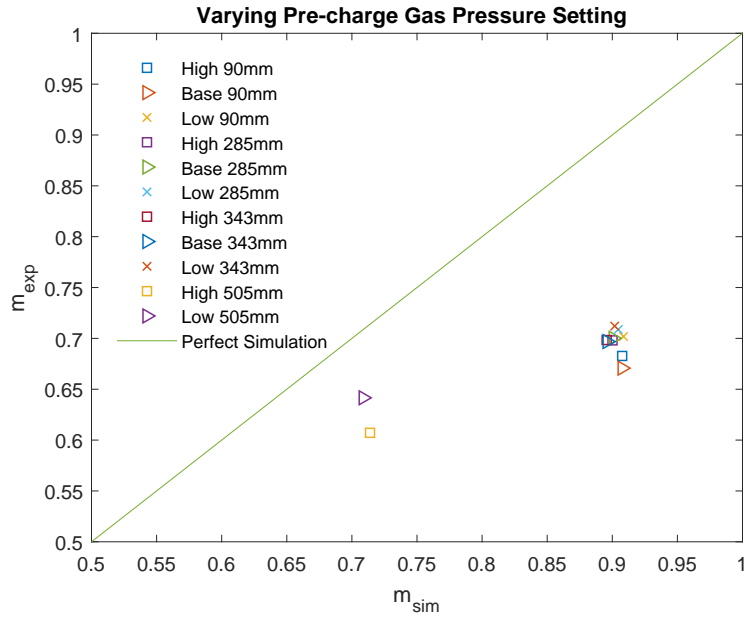


Figure 19: Simulated vs Experimental Fractional Coverage for PC with Varying Pre-charge Gas Pressure Setting where High = 500psi, Base = 325psi, and Low = 175psi.

4.2.4 Effect of Piston Speed

Lastly, the effect of varying piston speed at different rowlengths was observed. Figure 20 shows the viscosity vs shear rate for each simulated point. All points seem to lie in the Newtonian region. Figure 21 shows a zoomed in version of the viscosity vs shear rate plot to easily determine where the points lie on the curve.

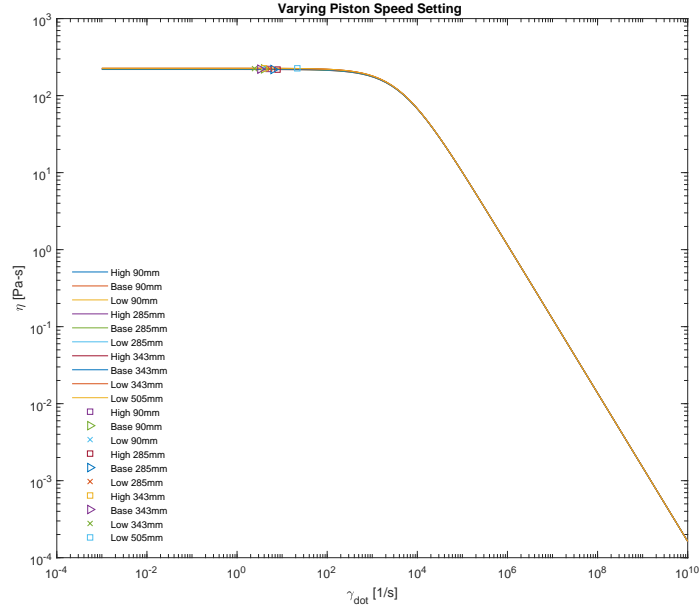


Figure 20: Viscosity vs Shear Rate for Each Simulated Point Varying Piston Speed Setting for PC where High = $10.0 \frac{in}{sec}$, Base = $3.1 \frac{in}{sec}$, and Low = $1.8 \frac{in}{sec}$.

Figure 22 shows the simulated vs experimental fractional coverages for each scenario. For rowlengths 90mm, 285mm, and 343mm, the points tend to be clustered around $m_{sim} \approx 0.9$. No obvious trends between the points at these rowlengths are noted. However, at 505mm, the simulation seems to predict the fractional coverage more accurately.

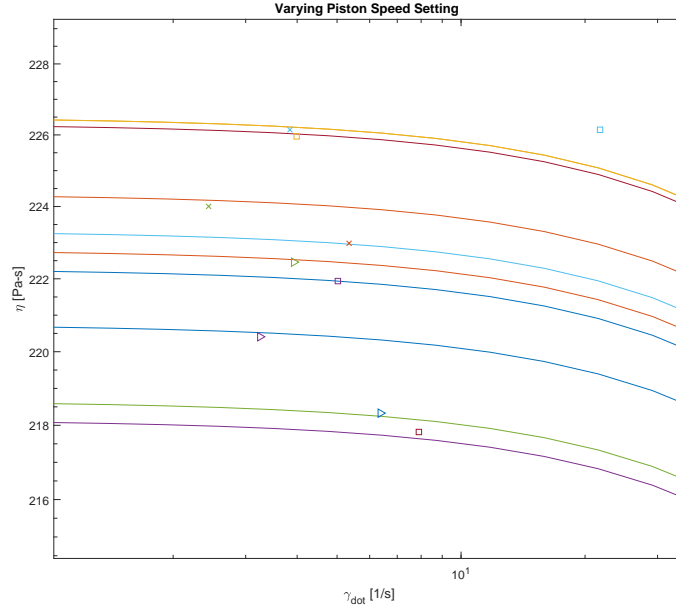


Figure 21: Zoomed Viscosity vs Shear Rate for Each Simulated Point Varying Piston Speed Setting for PC where High = $10.0 \frac{in}{sec}$, Base = $3.1 \frac{in}{sec}$, and Low = $1.8 \frac{in}{sec}$.

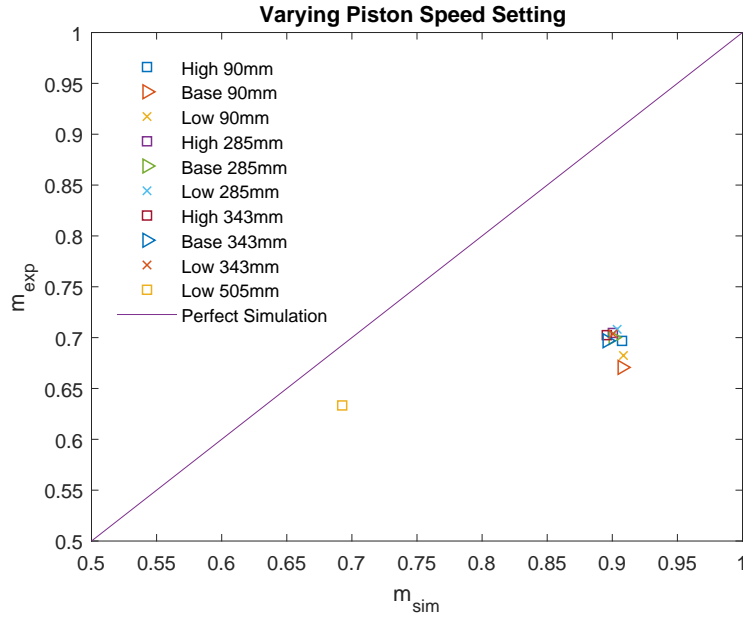


Figure 22: Simulated vs Experimental Fractional Coverage for PC with Varying Piston Speed Setting where High = $10.0 \frac{in}{sec}$, Base = $3.1 \frac{in}{sec}$, and Low = $1.8 \frac{in}{sec}$.

4.3 Simulation Results for Non-Newtonian Fluids under Non-Isothermal Conditions

4.3.1 Temperature Profiles

The temperature profile was calculated for polystyrene in the spiral mold cavity and can be seen in Figure 23. Though three viscosity models were used to simulate the behavior of this material, only one temperature profile is shown since the viscosity model had no influence on the temperature profile. The calculated temperature profiles show the non-isothermal condition of the system as time goes on as well as the cooling of the fluid in the mold cavity.

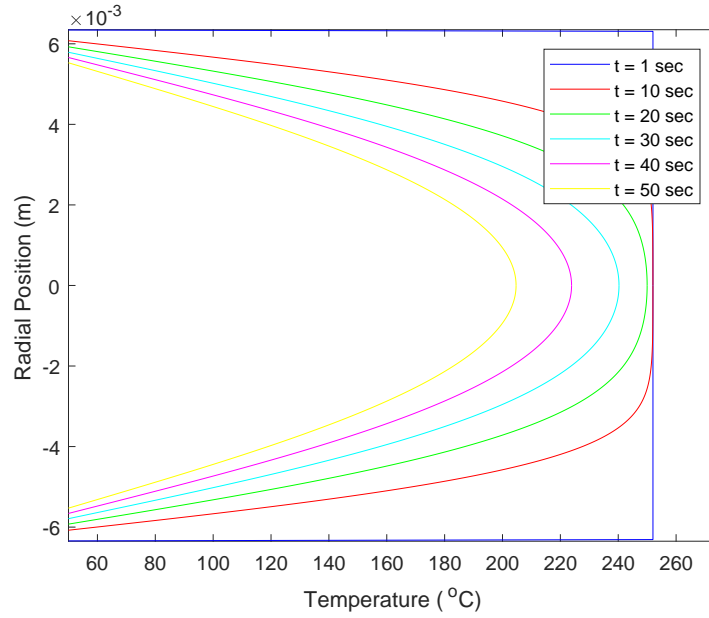


Figure 23: Temperature Profile for the Spiral Mold Cavity for PS.

4.3.2 Velocity Profiles

Normalized velocity profiles were also calculated for polystyrene for the cross-exponential, Ellis, and power law models. Figure 24 shows the normalized velocity profile calculated for polystyrene using the cross-exponential model. Again, non-isothermal velocity profiles are observed. The normalized velocity profiles show an increase in acceleration of the bubble as time increases as expected.

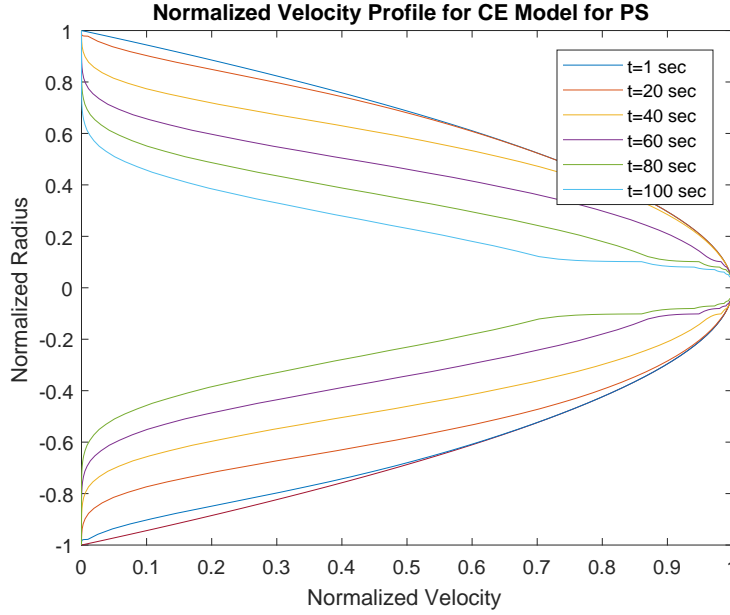


Figure 24: Normalized Velocity Profile for PS with Cross-Exponential Viscosity Model.

Figure 25 shows calculated normalized velocity profiles for polystyrene at $\tau_{wall} = 3.42 \text{ Pa} \cdot \text{s}^2$. Again, non-isothermal behavior and an increase in acceleration are observed.

Lastly, Figure 26 shows the calculated normalized velocity profile for polystyrene using the Power Law model. Non-isothermal behavior and increasing acceleration are observed.

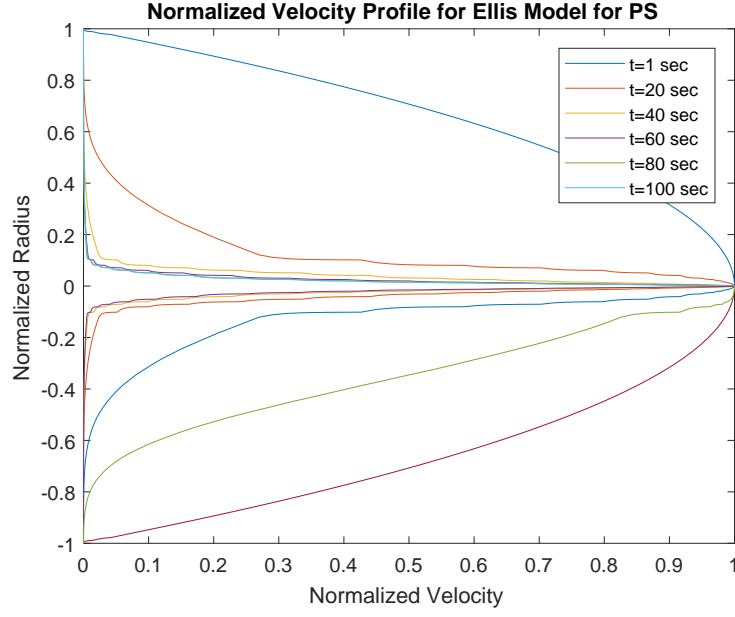


Figure 25: Normalized Velocity Profile for PS with Ellis Viscosity Model at $\tau_{wall} = 3.42Pa - s^2$.

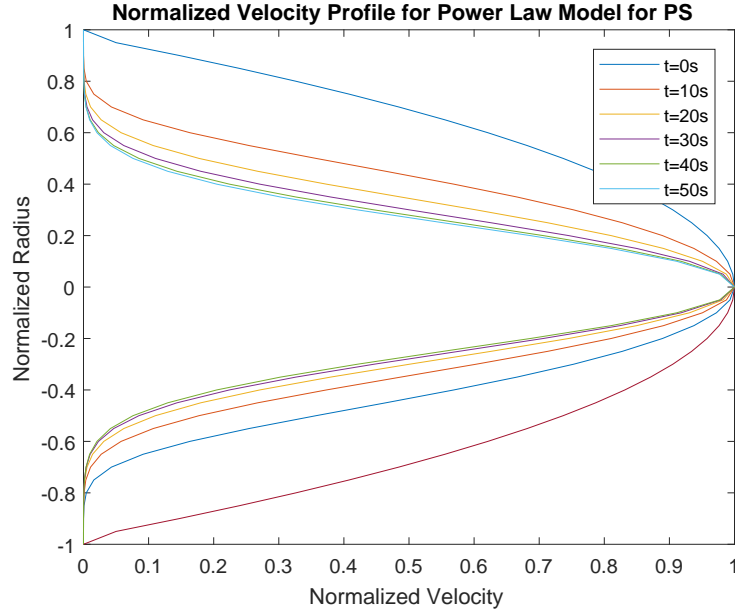


Figure 26: Normalized Velocity Profile for PS with Power Law Model.

4.3.3 m vs Fo at Base Conditions

Similar to polycarbonate, the simulated fractional coverage at base conditions was plotted against all experimental data to obtain an estimate of the simulated vs experimental results. m vs Fo plots were obtained for both the cross-exponential and Ellis models. The analysis was begun primarily by comparing the cross-exponential and Ellis models. The incorporation of the power law model was considered after.

Figure 27 shows m vs Fo for polystyrene using the cross-exponential model to describe the viscosity of the fluid. Figure 28 shows m vs Fo for polystyrene using the Ellis model to describe viscosity. Both plots tend to describe the same behavior at low Fo numbers but the Ellis model describes the elbow region of the data much more accurately than the cross-exponential model. Therefore, the Ellis model will be used for the comparison of simulated vs experimental fractional coverages.

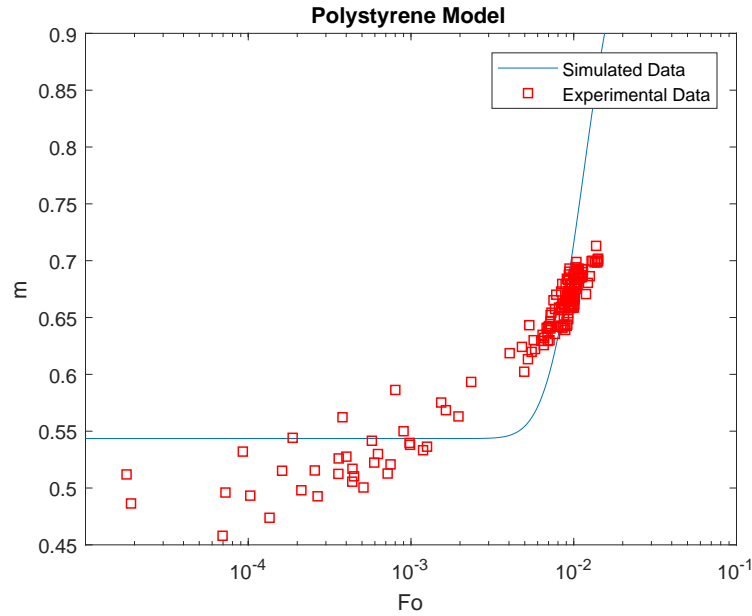


Figure 27: m vs Fo for PS with Cross-Exponential Viscosity Model at Base Processing Conditions.

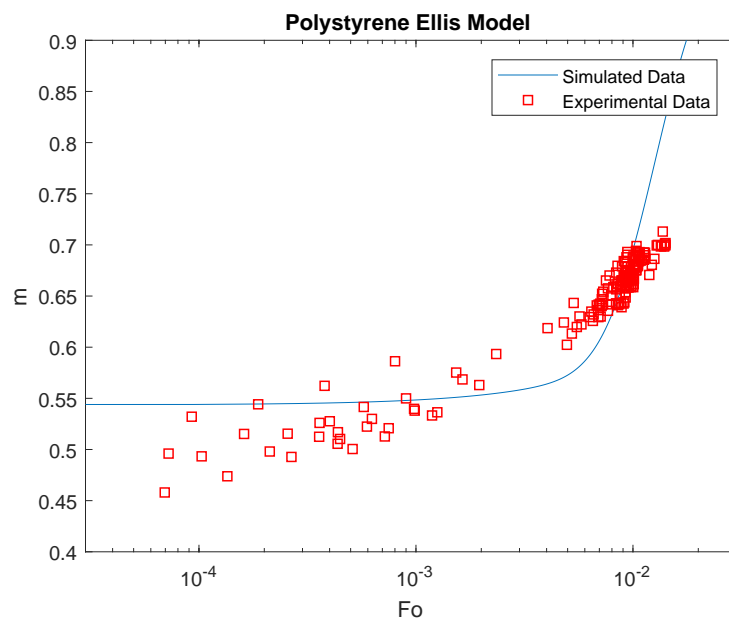


Figure 28: m vs Fo for PS with Ellis Viscosity Model at Base Processing Conditions.

4.4 Comparison of Simulation and Experimental Results for Fractional Coverage of non-Newtonian Fluids: Ellis Model

4.4.1 Effect of Shot Size

The first varied setting examined for polystyrene was shot size. Again, similar to the analysis of polycarbonate, viscosity and shear rate were determined and plotted. Figure 29 shows the viscosity vs shear rate plots for each scenario. All simulations lie in the shear-thinning region of the line, confirming our assumption that the polymer melt behaves as a non-Newtonian fluid. Figure 30 shows a zoomed in version of the viscosity vs shear rate plot for clarity.

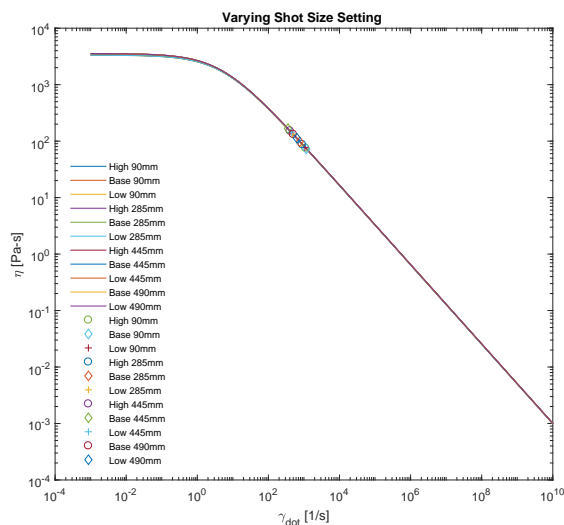


Figure 29: Viscosity vs Shear Rate for Each Simulated Point Varying Shot Size Setting Using Ellis Model for PS where High = 3.15in, Base = 3.0in, and Low = 2.85in.

Figure 31 shows the simulated vs experimental fractional coverages for polystyrene with varying shot size setting. For scenarios at low to middle rowlengths (90mm, 285mm), the simulation extremely overpredicts the fractional coverage. As rowlength increases, the simulation tends to predict a more accurate fractional coverage.

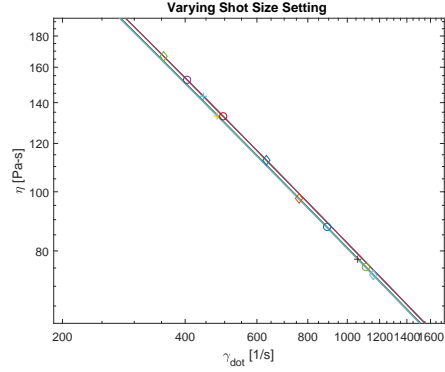


Figure 30: Zoomed Viscosity vs Shear Rate for Each Simulated Point Varying Shot Size Setting Using Ellis Model for PS where High = 3.15in, Base = 3.0in, and Low = 2.85in.

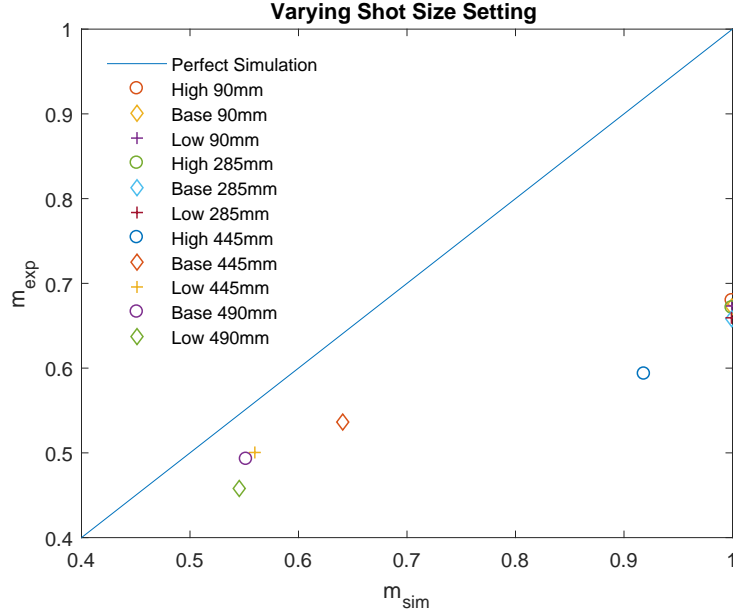


Figure 31: Simulated vs Experimental Fractional Coverage for PS with Varying Shot Size Setting Using Ellis Model where High = 3.15in, Base = 3.0in, and Low = 2.85in.

4.4.2 Effect of Delay Time

The next observed effect was the variation of delay time setting. Again, viscosity vs shear rate was plotted. Figure 32 shows the viscosity vs shear rate for polystyrene with varying delay time setting. We observe that all examined points lie in the shear-thinning portion of the curve suggesting the polystyrene behaved as a non-Newtonian fluid. Figure 33 shows a zoomed in version of the viscosity vs shear rate plot.

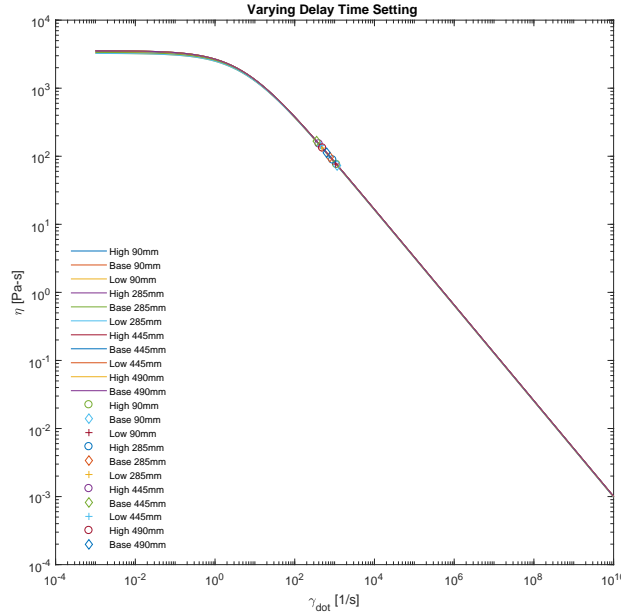


Figure 32: Viscosity vs Shear Rate for Each Simulated Point Varying Delay Time Setting Using Ellis Model for PS where High = 4.37sec, Base = 2.37sec, and Low = 0.62sec.

Figure 34 shows the simulated vs experimental fractional coverages for each tested condition. Similar to the shot size setting, the simulation highly overpredicts the fractional coverage at 90mm and 285mm but provides more accurate predictions at 445mm and 490mm.

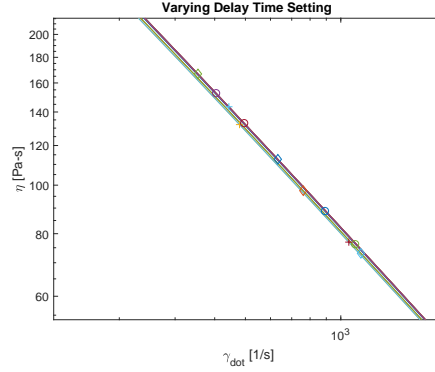


Figure 33: Zoomed Viscosity vs Shear Rate for Each Simulated Point Varying Delay Time Setting Using Ellis Model for PS where High = 4.37sec, Base = 2.37sec, and Low = 0.62sec.

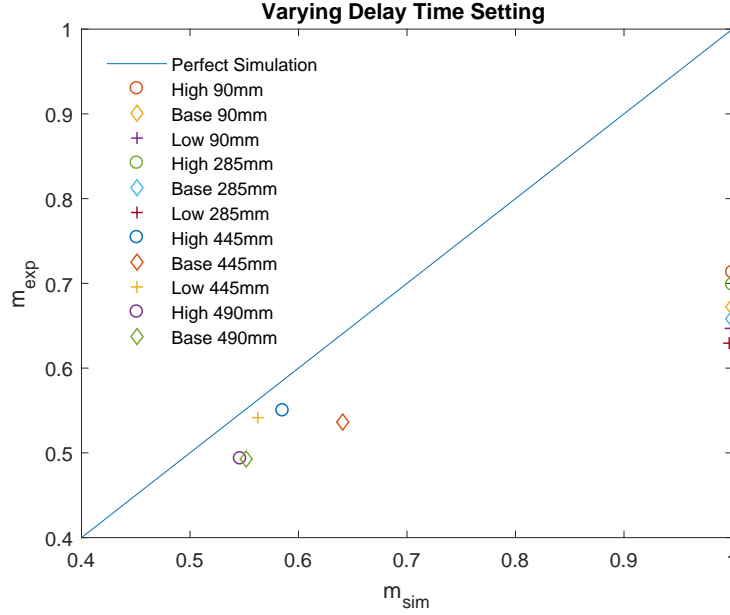


Figure 34: Simulated vs Experimental Fractional Coverage for PS with Varying Delay Time Setting Using Ellis Model where High = 4.37sec, Base = 2.37sec, and Low = 0.62sec.

4.4.3 Effect of Pre-charge Gas Pressure

The pre-charge gas pressure setting was then varied. Non-Newtonian behavior was noticed with the viscosity vs shear rate plot in Figure 35 for each data point. Figure 36 shows a zoomed-in version of the viscosity vs shear rate plot to observe the subtle differences in temperature between each scenario.

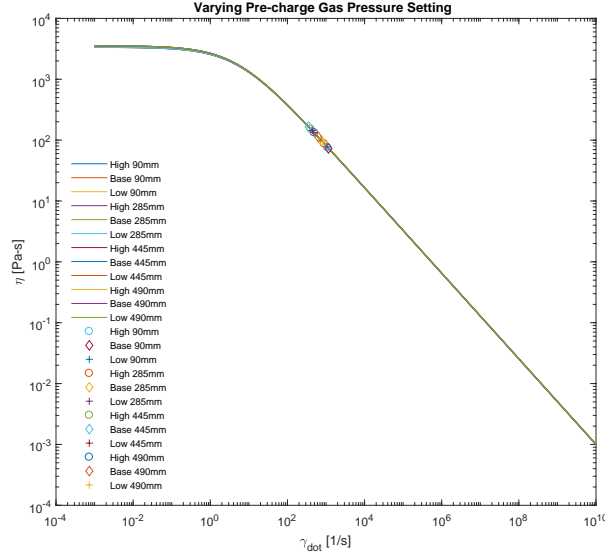


Figure 35: Viscosity vs Shear Rate for Each Simulated Point Varying Pre-charge Gas Pressure Setting Using Ellis Model for PS where High = 500psi, Base = 350psi, and Low = 200psi.

Figure 37 shows the experimental vs simulated fractional coverages for polystyrene with varying pre-charge gas pressure setting. Again, at rowlengths 90mm and 285mm the simulation overpredicts the fractional coverage. At rowlengths 445mm and 490mm the predictions become more accurate. We can also observe that for the low pre-charge gas pressure setting at 490mm the simulation prediction is almost the same (slightly lower) as the experimental value.

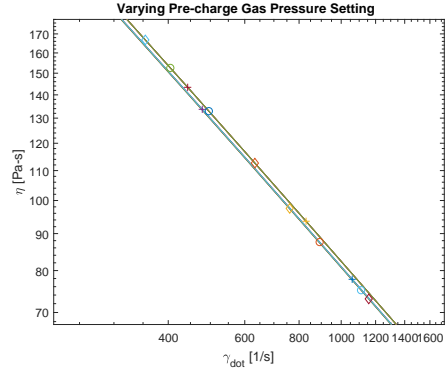


Figure 36: Zoomed Viscosity vs Shear Rate for Each Simulated Point Varying Pre-charge Gas Pressure Setting Using Ellis Model for PS where High = 500psi, Base = 350psi, and Low = 200psi.

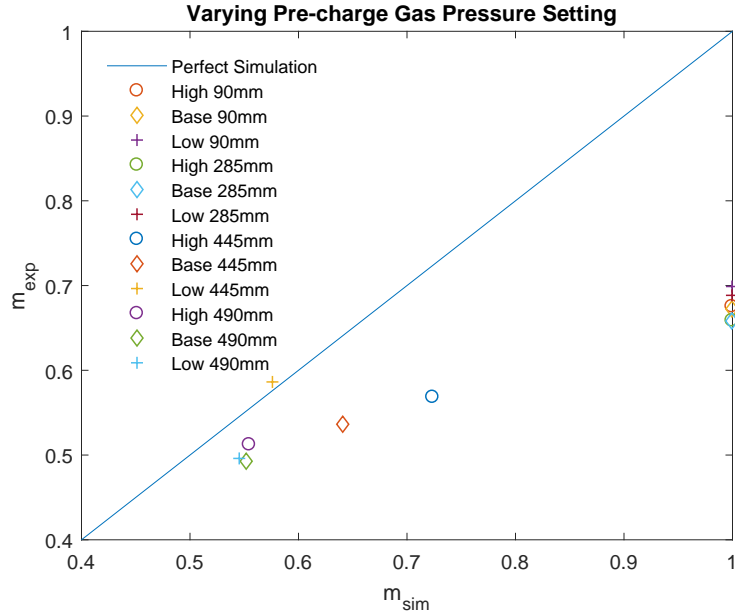


Figure 37: Simulated vs Experimental Fractional Coverage for PS with Varying Pre-charge Gas Pressure Setting Using Ellis Model where High = 500psi, Base = 350psi, and Low = 200psi.

4.4.4 Effect of Piston Speed

The piston speed setting was then varied. Figure 38 confirms the assumption that polystyrene behaved as a non-Newtonian fluid. Figure 39 shows a zoomed-in version of the viscosity vs shear rate plot.

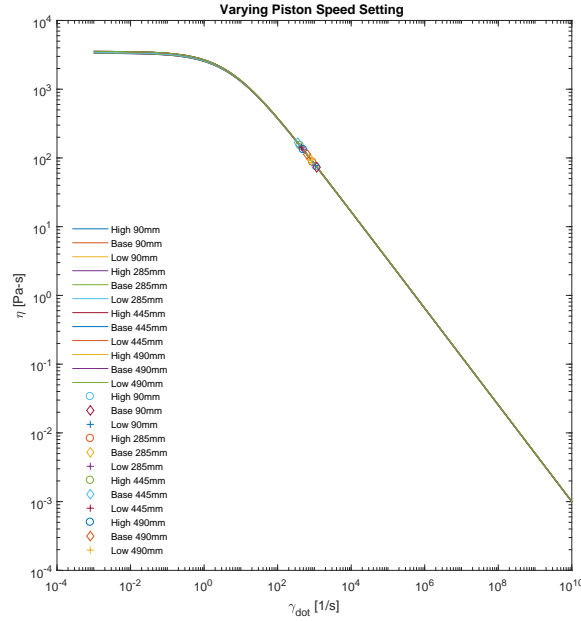


Figure 38: Viscosity vs Shear Rate for Each Simulated Point Varying Piston Speed Setting Using Ellis Model for PS where High = $10.0 \frac{in}{sec}$, Base = $3.1 \frac{in}{sec}$, and Low = $1.8 \frac{in}{sec}$.

Figure 40 shows the simulated vs experimental fractional coverages for polystyrene with varying piston speed setting at several rowlengths. The simulation does not predict well the fractional coverage at rowlengths 90mm and 285mm for all piston speed settings. Rowlength 490mm simulated the most accurate prediction for all piston speed settings.

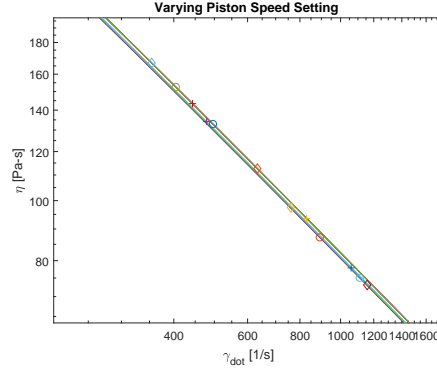


Figure 39: Zoomed Viscosity vs Shear Rate for Each Simulated Point Varying Piston Speed Setting Using Ellis Model for PS where High = $10.0 \frac{in}{sec}$, Base = $3.1 \frac{in}{sec}$, and Low = $1.8 \frac{in}{sec}$.

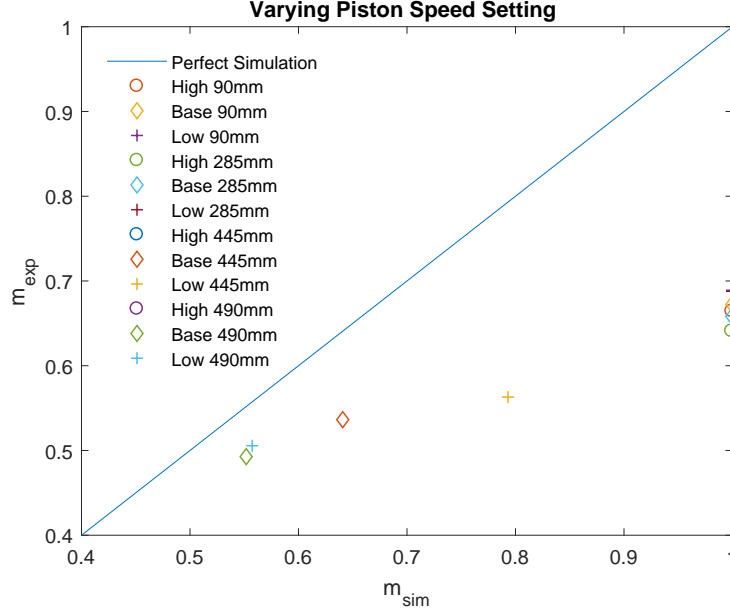


Figure 40: Simulated vs Experimental Fractional Coverage for PS with Varying Piston Speed Setting Using Ellis Model where High = $10.0 \frac{in}{sec}$, Base = $3.1 \frac{in}{sec}$, and Low = $1.8 \frac{in}{sec}$.

4.4.5 Effect of Melt Temperature

The last varied setting examined was melt temperature. Figure 41 shows the viscosity vs shear rate plot for each scenario. It is interesting to observe that due to the variation of melt temperature, the viscosity vs shear rate lines shift into three different clustered lines due to the three settings of melt temperature tested. Again we observe the polystyrene behaved as a non-Newtonian fluid for all scenarios. Figure 42 shows a zoomed-in version of Figure 43.

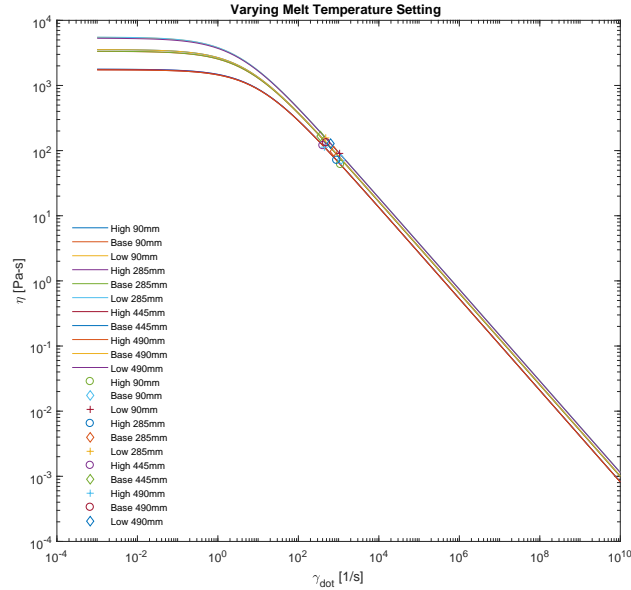


Figure 41: Viscosity vs Shear Rate for Each Simulated Point Varying Melt Temperature Setting Using Ellis Model for PS where High = 511.7°F, Base = 485.6°F, and Low = 471.5°F.

Figure 43 shows the simulated vs experimental fractional coverages for polystyrene at different rowlengths and melt temperature settings. The simulation does not predict the fractional coverage at 90mm and 285mm well but predicts the fractional coverages at 490mm almost exactly for all melt temperature settings. The simulated fractional coverages at 445mm were much more accurately predicted compared to 90mm and 285mm but are not exact.

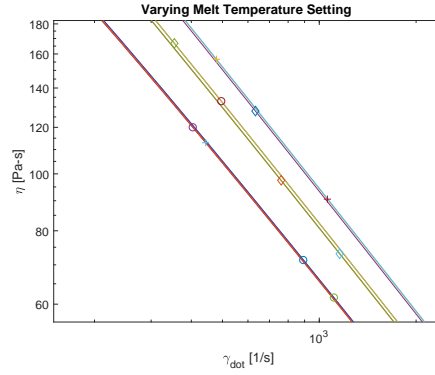


Figure 42: Zoomed Viscosity vs Shear Rate for Each Simulated Point Varying Melt Temperature Setting Using Ellis Model for PS where High = 511.7°F, Base = 485.6°F, and Low = 471.5°F.

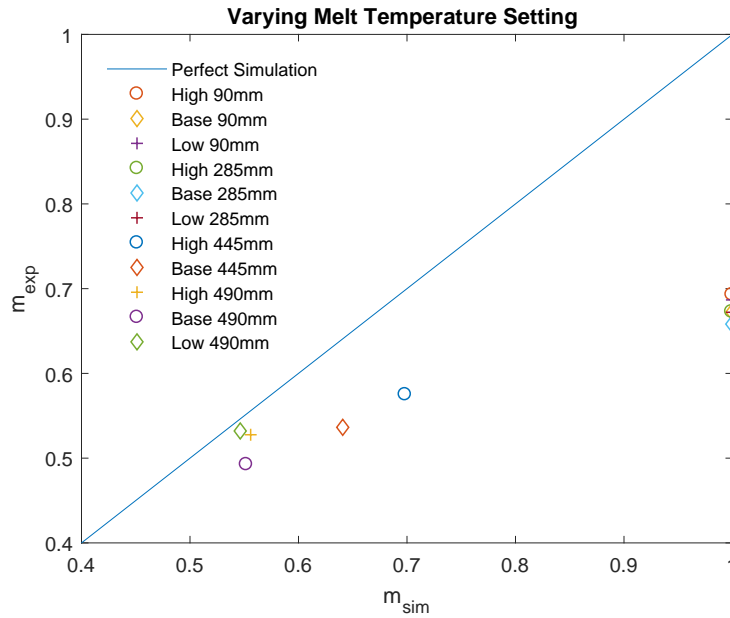


Figure 43: Simulated vs Experimental Fractional Coverage for PS with Varying Melt Temperature Setting Using Ellis Model where High = 511.7°F, Base = 485.6°F, and Low = 471.5°F.

4.5 Comparison of Simulation and Experimental Results for Fractional Coverage of non-Newtonian Fluids: Power Law Model

After obtaining the results using the Ellis model, the Power Law model was used to attempt to obtain more accurate simulation responses at lower rowlengths. In this section, viscosity vs shear rate was not plotted due to the power law behavior noted in the previous sections of this work. Rowlengths of 90mm, 285mm, 445mm, and 490mm were chosen, the same as those used for the Ellis model.

4.5.1 Effect of Shot Size

The first varied setting considered was shot size. Figure 44 shows the obtained results using the power law model for varying shot size setting. All simulated results are higher than the experimental results. At rowlengths of 90mm and 285mm, m_{sim} lies around 0.9 for all simulations, at least 0.2 higher than any of the experimental results. A higher fractional coverage prediction would infer a higher bubble radius prediction. This could indicate that the simulated heat transfer is occurring faster compared to reality. Similar trends appear with 445mm and 490mm rowlengths. m_{sim} is at least 0.1 higher than the experimental results.

4.5.2 Effect of Delay Time

The effect of delay time was then observed. Figure 45 shows the obtained results using the power law model while varying the delay time setting. Again m_{sim} tends to be at least 0.1 higher than the experimental results. It is also interesting to note for this setting that the high, base, and low settings at 90mm and 285mm are predicted to be almost exactly the same values despite their differences in rowlengths.

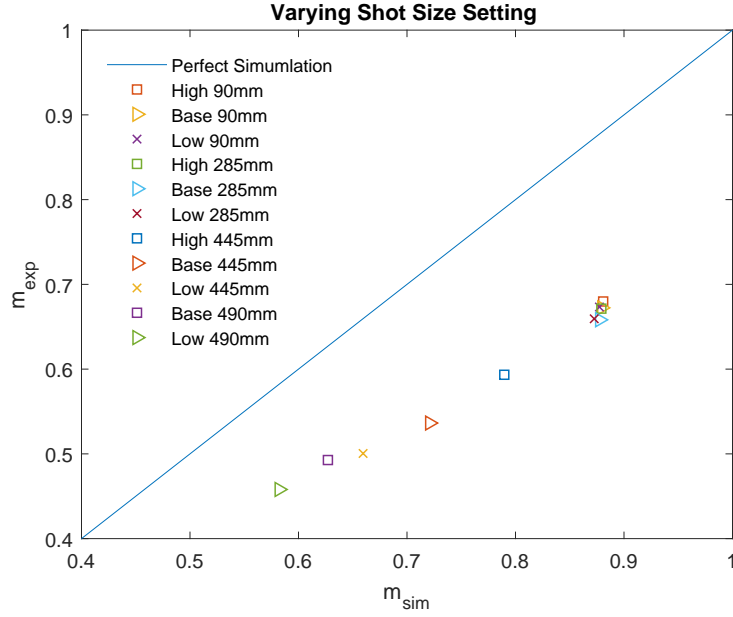


Figure 44: Simulated vs Experimental Fractional Coverage for PS Using the Power Law Model with Varying Shot Size Setting where High = 3.15in, Base = 3.0in, and Low = 2.85in.

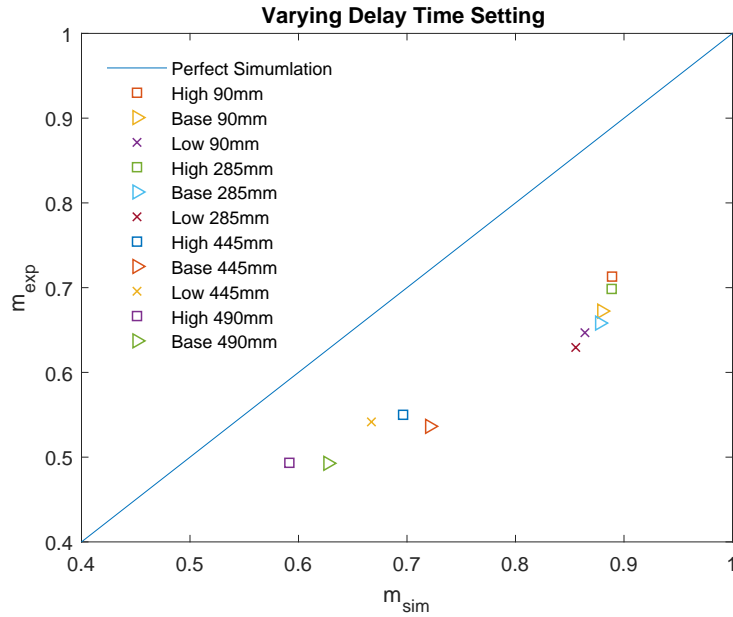


Figure 45: Simulated vs Experimental Fractional Coverage for PS Using the Power Law Model with Varying Delay Time Setting where High = 4.37sec, Base = 2.37sec, and Low = 0.62sec.

4.5.3 Effect of Pre-charge Gas Pressure

Figure 46 shows the obtained results using the power law model while varying the pre-charge gas pressure setting. When observing the varying effects of this setting, m_{sim} is exactly the same at 90mm and 285mm for all settings. This is not true for either 445mm or 490mm. Additionally, the predictions of this model tend to be 0.1-0.3 greater than the experimental fractional coverages.

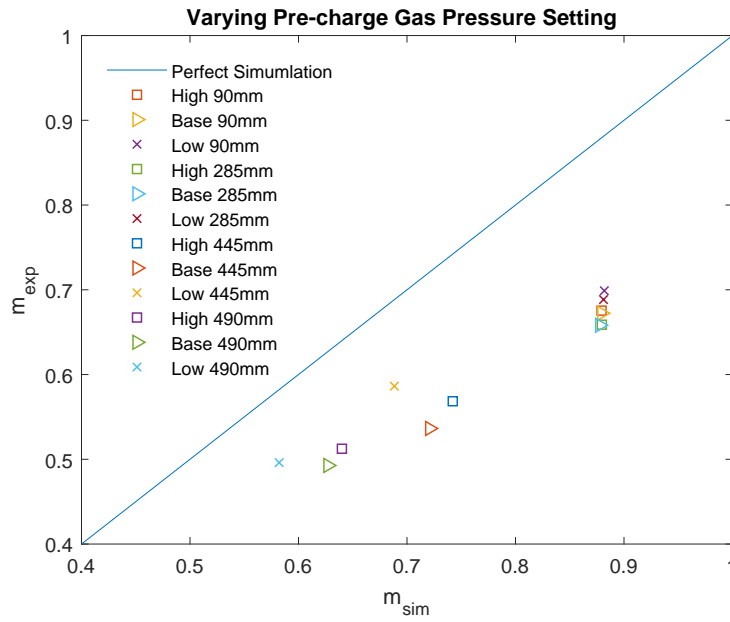


Figure 46: Simulated vs Experimental Fractional Coverage for PS Using the Power Law Model with Varying Pre-charge Gas Pressure Setting where High = 500psi, Base = 350psi, and Low = 200psi.

4.5.4 Effect of Piston Speed

Next, the effect of varying piston speed setting was observed. Figure 47 shows the simulated vs experimental fractional coverages while varying piston speed setting. It should be noted that no experimental results were obtained for the High setting at 445mm or 490mm rowlengths. Again, $m_{sim} \approx 0.9$ for all settings at 90mm and 285mm. This fact is not true at 445mm and 490mm but m_{sim} is still overpredicted for all rowlengths and all settings.

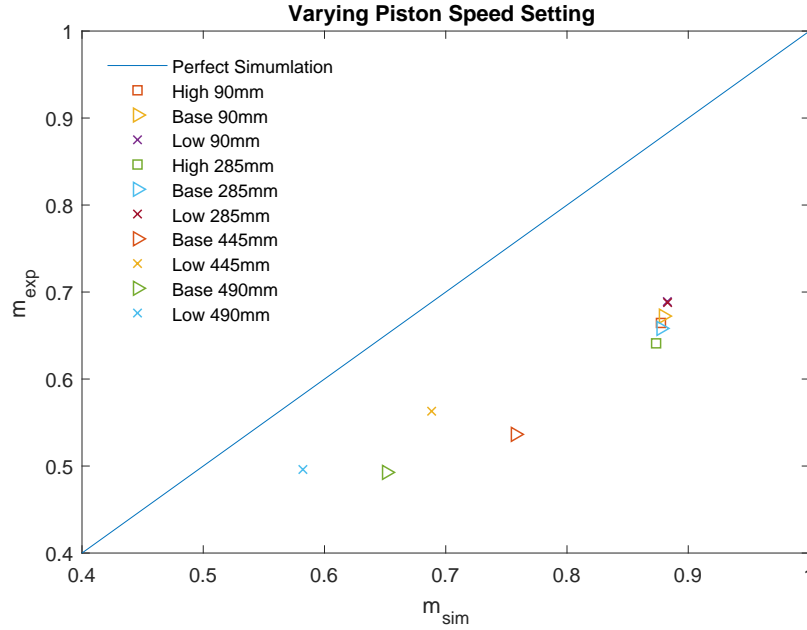


Figure 47: Simulated vs Experimental Fractional Coverage for PS Using the Power Law Model with Varying Piston Speed Setting where High = $10 \frac{in}{sec}$, Base = $3.1 \frac{in}{sec}$, and Low = $2.8 \frac{in}{sec}$.

4.5.5 Effect of Melt Temperature

Lastly, the effect of melt temperature was observed. Figure 48 shows the simulated vs experimental fractional coverages for polystyrene with varying melt temperature setting. Simulated fractional coverage overpredicts fractional coverage for all settings and rowlengths tested. Similar to other varied variables, simulated fractional coverage for 90mm and 285mm rowlengths are exactly the same. While this does not hold true for 445mm and 490mm, m_{sim} is still 0.1-0.3 higher than m_{exp} .

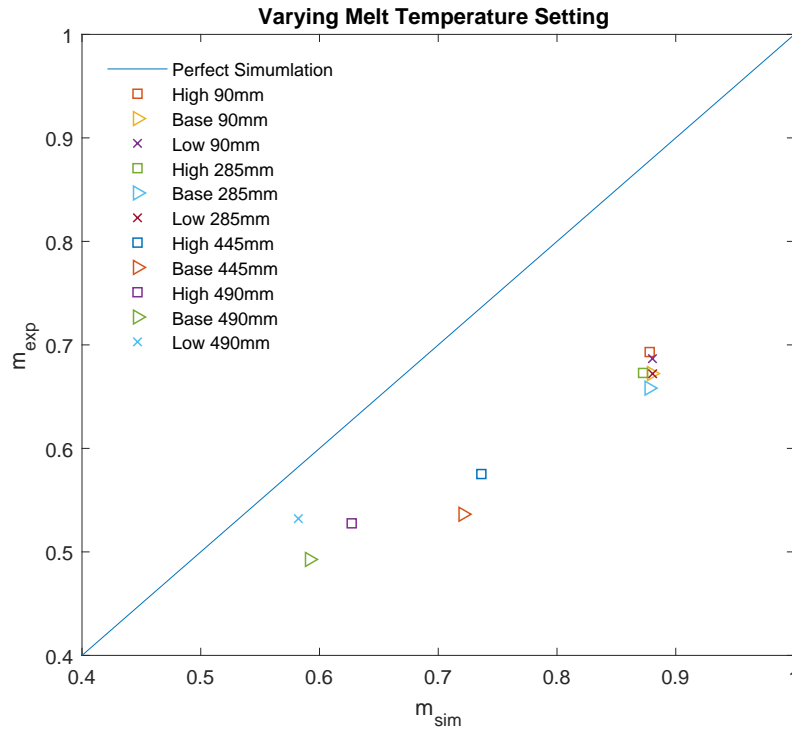


Figure 48: Simulated vs Experimental Fractional Coverage for PS Using the Power Law Model with Varying Melt Temperature Setting where High = 511.7°F, Base = 485.6°F, and Low = 471.5°F.

4.6 Comparison of Simulation Results for non-Newtonian Fluids under Non-Isothermal Conditions

The simulated results from both the Ellis and Power Law models were then plotted jointly to compare the predictions of the two models for each varied setting and rowlength tested.

4.6.1 Effect of Shot Size

Figure 49 shows the simulated fractional coverages using both the Ellis and power law models against experimental fractional coverage. Overall, the Ellis model predicts fractional coverage more accurately at 445mm and 490mm while the power law model predicts fractional coverage more accurately at 90mm and 285mm. This is not to say that their predictions are precise but rather to highlight the advantages of each model.

4.6.2 Effect of Delay Time

Figure 50 shows the simulated fractional coverages using both the Ellis and power law models against experimental fractional coverage. Again, the simulation utilizing the Ellis model predicts fractional coverage at 445mm and 490mm more accurately than the simulation utilizing the power law model while the power law model simulation predicts fractional coverage more accurately an 90mm and 285mm than the Ellis model simulation.

4.6.3 Effect of Pre-charge Gas Pressure

Figure 51 plots simulated fractional coverage vs experimental fractional coverage for both the Ellis model and power law model simulations. The Ellis model simulation more accurately predicts the fractional coverage at the higher rowlengths while the power law model more accurately predicts the fractional coverage at lower rowlengths.

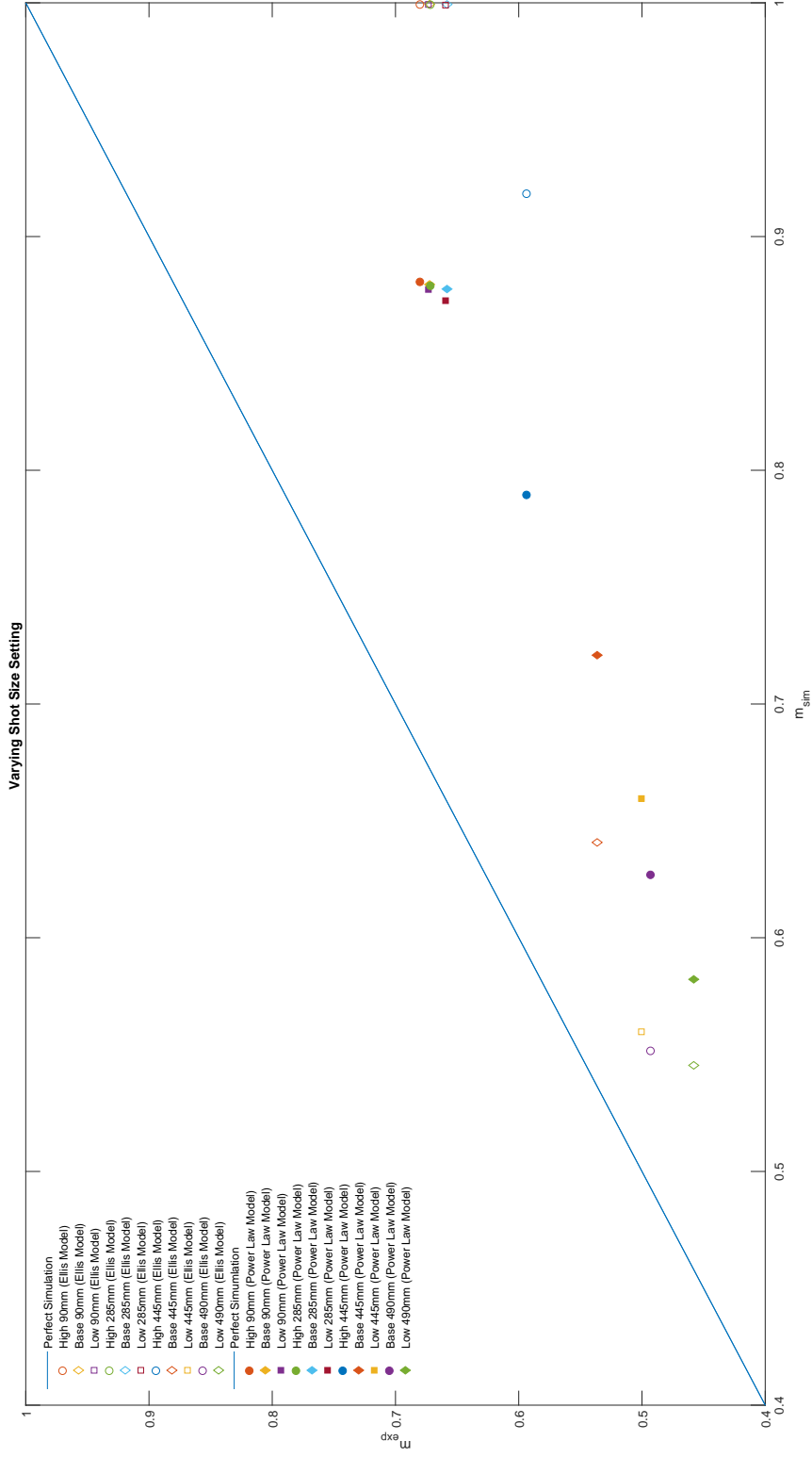


Figure 49: Comparison of Simulated vs Experimental Fractional Coverage for PS of Both the Ellis and Power Law Models with Varying Shot Size Setting where High = 3.15in, Base = 3.0in, and Low = 2.85in.

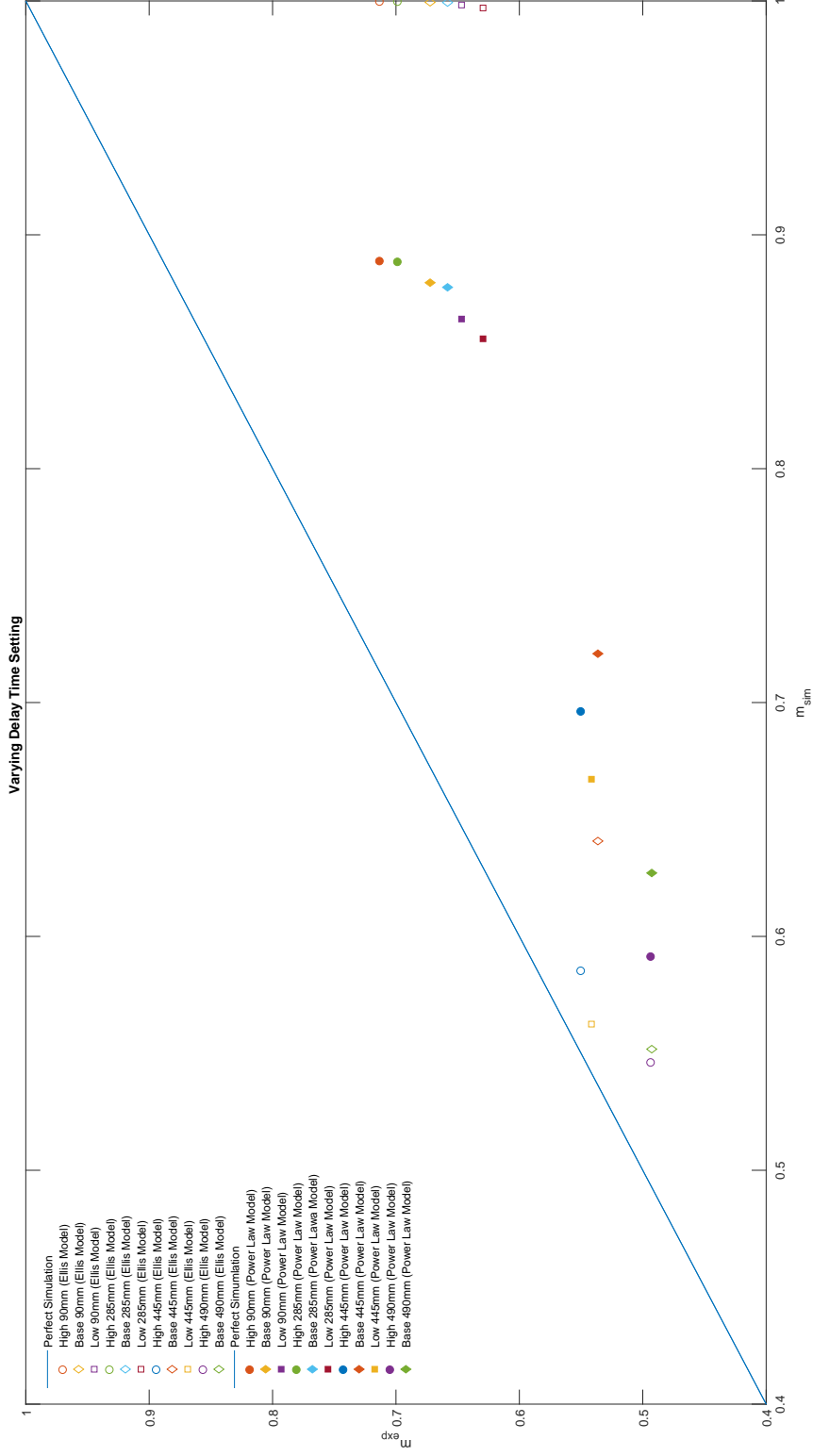


Figure 50: Comparison of Simulated vs Experimental Fractional Coverage for PS of Both the Ellis and Power Law Models with Varying Delay Time Setting where High = 4.37sec, Base = 2.37sec, and Low = 0.62sec.

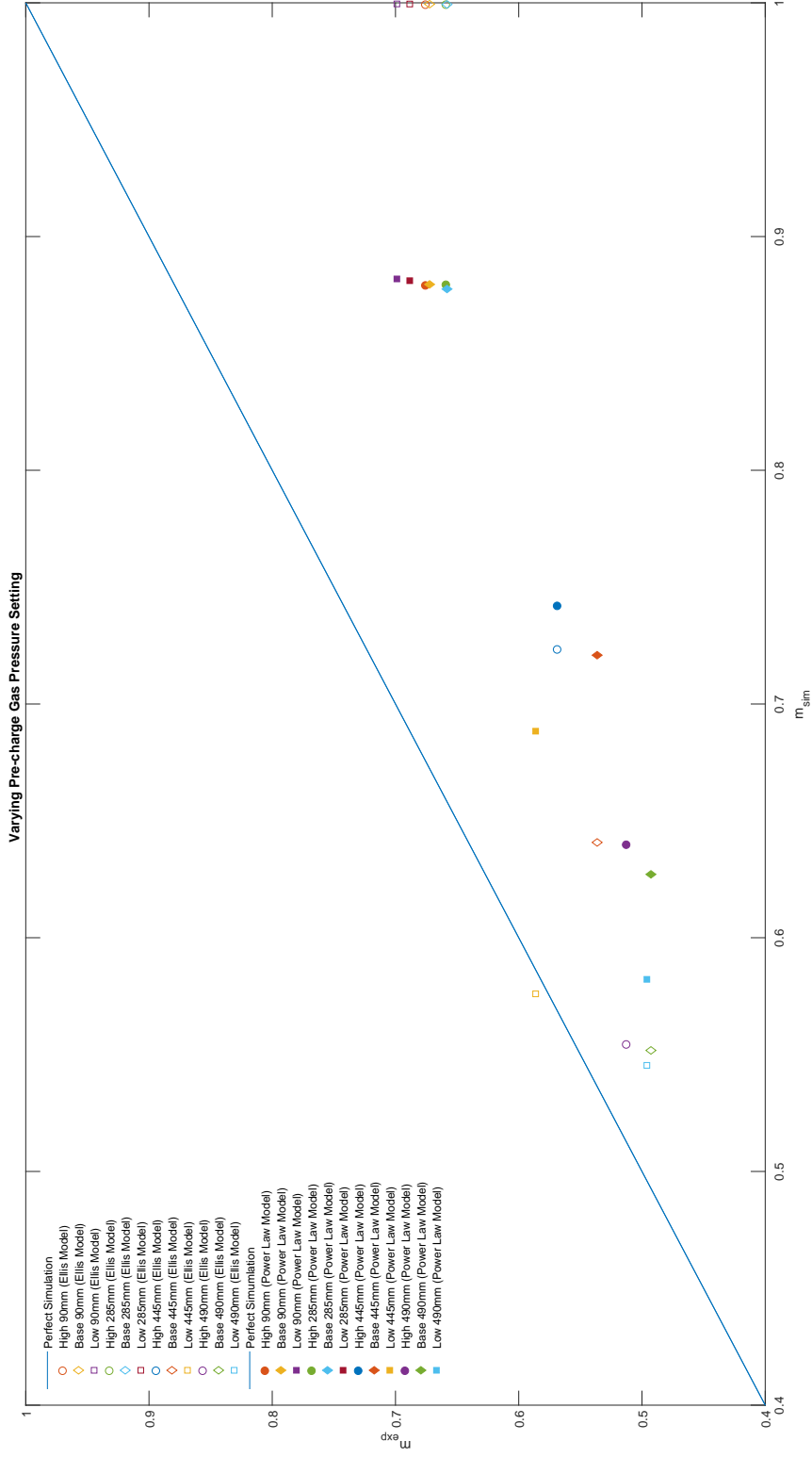


Figure 51: Comparison of Simulated vs Experimental Fractional Coverage for PS of Both the Ellis and Power Law Models with Varying Pre-charge Gas Pressure Setting where High = 500psi, Base = 350psi, and Low = 200psi.

4.6.4 Effect of Piston Speed

Figure 52 shows the simulated fractional coverages using both the Ellis and power law models against experimental fractional coverage. Once again, the Ellis model simulation is more accurate at 445mm and 490mm while the power law model is more accurate at 90mm and 285mm.

4.6.5 Effect of Melt Temperature

Figure 53 plots the experimental fractional coverage vs the simulated fractional coverages calculated by using either the power law model or the Ellis model. The power law model more accurately predicts fractional coverage at 90mm and 285mm compared to the Ellis model while the Ellis model more accurately predicts fractional coverage at 445mm and 490mm compared to the power law model simulations. The predicted fractional coverages are almost always higher than the experimental values which could indicate a flaw in the simulation model, whether it be an incorrect assumption or relation.

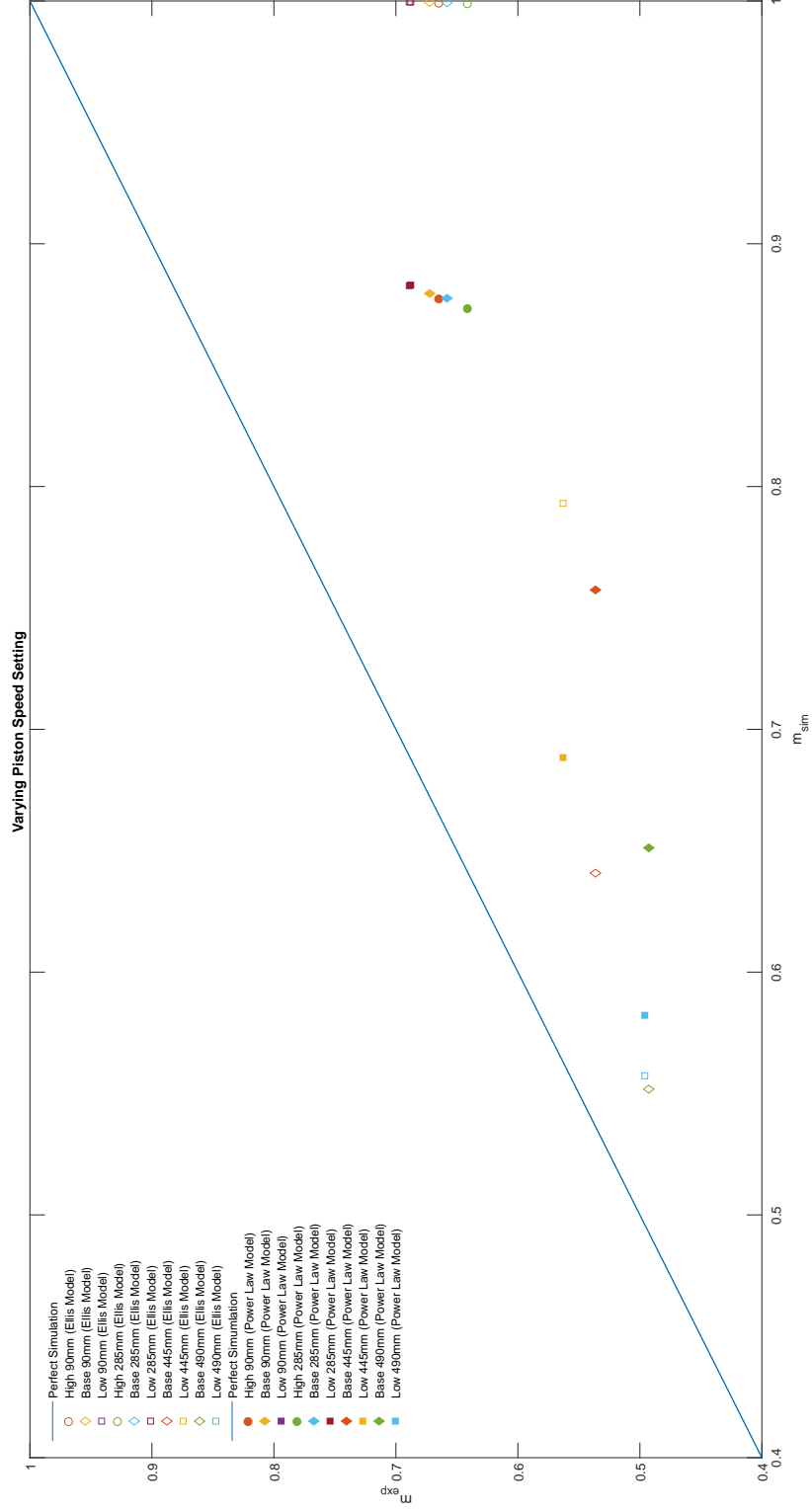


Figure 52: Comparison of Simulated vs Experimental Fractional Coverage for PS of Both the Ellis and Power Law Models with Varying Shot Size Setting where High = $10 \frac{in}{sec}$, Base = $3.1 \frac{in}{sec}$, and Low = $2.8 \frac{in}{sec}$.

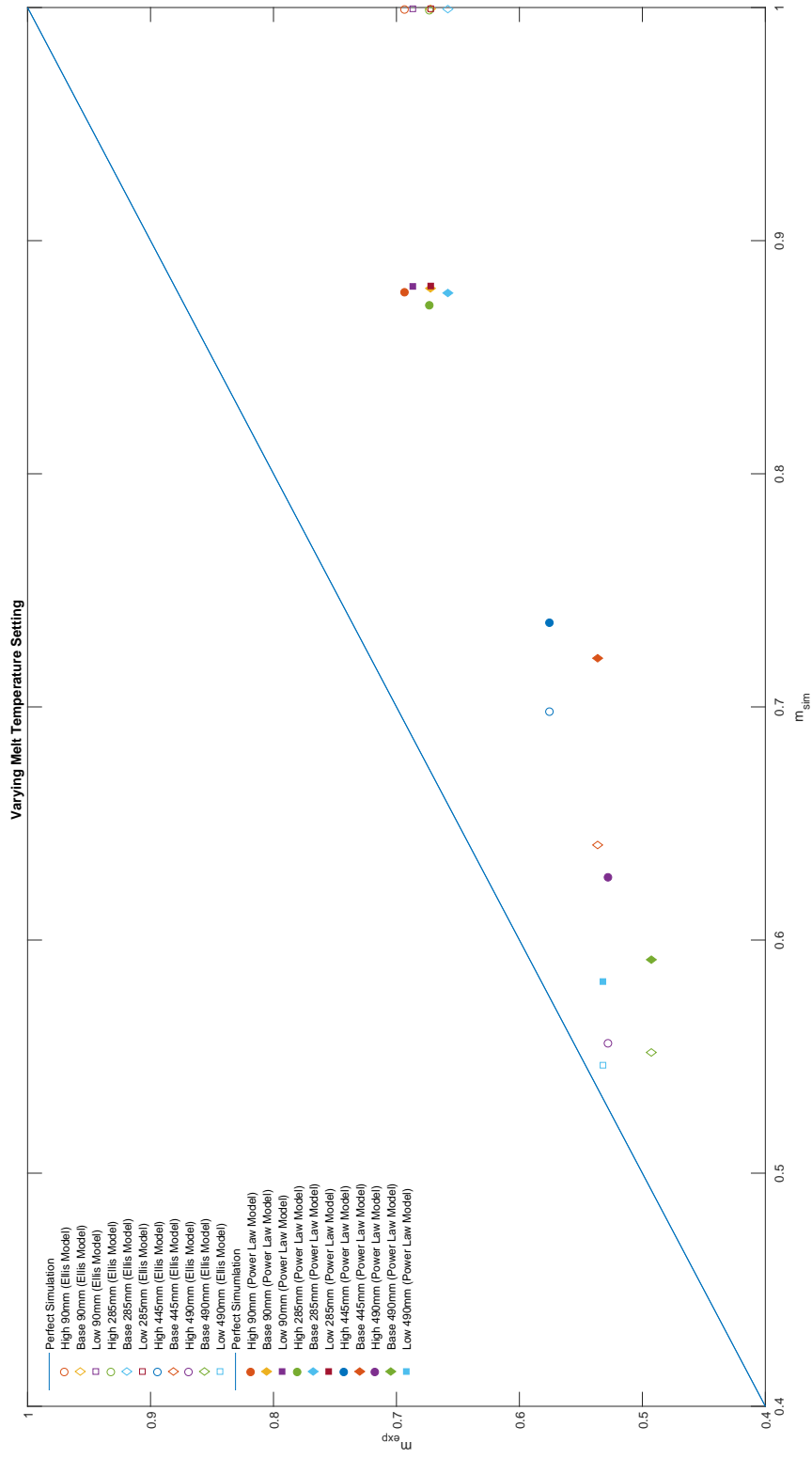


Figure 53: Comparison of Simulated vs Experimental Fractional Coverage for PS of Both the Ellis and Power Law Models with Varying Melt Temperature Setting where High = 511.7°F, Base = 485.6°F, and Low = 471.5°F.

5 Conclusion

Three major simulation models for predicting fractional coverage using gas-assisted injection of polymer melts in tube geometries were presented. The first model presented was for fluids following Newtonian behavior. The cross-exponential zero-shear viscosity model was used and the determined fractional coverage values were compared with experimental data collected by Kaminski [9]. Viscosity vs shear rate plots were obtained for all tested scenarios and showed that the assumption that polycarbonate behaved as a Newtonian fluid was valid. The comparison of the simulated and experimental data showed good predictions at further rowlengths than those closer to the entrance. In addition, the variation of several process parameters were examined. No major trends were found between the variation of settings for each parameter. Further, velocity profiles and temperature profiles were calculated and plotted and the non-isothermal behavior was shown to be captured by the system as well as the cooling of the tube and narrowing bubble as time passed.

The non-Newtonian modeling began by the determination of temperature and velocity profiles using both the Ellis and cross-exponential viscosity models. Both models showed non-isothermal behavior as time passed as well as fluid cooling and the narrowing of the bubble through the tube. It was observed that at the base conditions for all data points, the Ellis model seemed to predict the fractional coverage more accurately than the cross-exponential model. Therefore, the Ellis model was used to determine simulated fractional coverages for several process parameters settings and rowlengths. Viscosity vs shear rate was plotted for all tested scenarios and it was determined that the polystyrene behaved as a non-Newtonian fluid during these experiments. The simulated vs experimental fractional coverage plots again showed better predictions as rowlength increased. Several process parameters were varied and examined when plotting the simulated vs experimental fractional coverages. No major trends between the levels of each setting were observed.

The Ellis model extremely overpredicted fractional coverage values at low rowlengths. The power law model was incorporated to attempt to obtain more accurate predictions at lower rowlengths compared to the Ellis model. With the incorporation of the power

law model, more accurate predictions of the fractional coverage at lower rowlengths were determined albeit still higher than a perfect simulation. Overall, for the non-Newtonian modeling, the Ellis model tended to be more accurate at higher rowlengths while the power law model was more accurate at lower rowlengths.

6 Future Work

The models presented take into consideration multiple assumptions that limit the prediction models. The most limiting assumption is that of high capillary number. This is presented in the calculation of asymptotic isothermal fractional coverage. The model also assumed forced convection during the calculation of the heat transfer coefficient. This model would not be valid in cases where free convection is the dominant form of heat transfer (i.e. a tube in a water bath). The models are also limited by the viscosity models chosen to model the fluids. If chosen fluids do not follow the viscosity model chosen, the model is no longer valid. In addition, the modeling does not take into consideration the effects of viscous heating. Lastly, the temperature dependence of viscosity models were taken at face value. Sensitivity studies on these temperature dependence of these models regarding the system could be completed.

The model could be expanded to eliminate the stated limitations. This work could also be expanded to examine the effect of capillary number on the system as well as the expansion of the model to incorporate different geometries. Studies regarding the effects of viscous heating on the system could also be determined as to whether this phenomena greatly affects the system or dissipates in the loading tip of the mold.

7 References

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8 Appendix A: Abbreviations and Notation

α : degree of shear-thinning behavior

α_p : thermal diffusivity of polymer melt

α_s : thermal diffusivity of steel

A_p : Cross-sectional area of polymer deposited on tube walls by gas bubble

A_t : Tube cross-sectional area

B: coefficient dependent on molecular weight and temperature

Ca: Capillary number

dr_2 : radial step change

ΔP : pressure gradient

η : Fluid viscosity

η_o : Zero Shear Viscosity

FDM: Finite Difference Method

FEM: Finite Element Method

Fo: Fourier number

GAIM: Gas-assisted Injection Molding

$\dot{\gamma}$: Shear rate

h: convective heat transfer coefficient

k_s : thermal conductivity of stainless steel

L: tube length

m: Fractional coverage

m_∞ : isothermal asymptotic fractional coverage

n: shear-thinning coefficient in cross-exponential model

n: power law index

Nu: Nusselt number

Pr: Prandtl number

PC: Polycarbonate

PS: Polystyrene

Q : volumetric flow rate

r_p : radial position of polymer

r_s : radial position of steel

Ra : Raleigh number

R_{out} : Radius of outer tube

R_t : Tube radius

R_x : Bubble radius

R_{01} : inner tube radius

σ : Fluid surface tension

τ_{rz} : shear stress

τ_{wall} : shear stress at the wall

$\tau_{1/2}$: value of shear stress at which the apparent viscosity has dropped to half its zero shear value

τ^* : shear-stress level in relation to the transition between the Newtonian and shear-thinning regimes

T_b : temperature sensitivity coefficient

T_B : temperature of bulk fluid

T_i : initial temperature of polymer melt

T_p : temperature of polymer

T_s : temperature of steel/water interface

T_w : temperature of the wall

u_z : Fluid velocity

u^* : parabolic velocity profile

U_b : Bubble velocity

9 Appendix B: MATLAB code for PC Modeling

11/1/17 7:35 AM C:\Users\estanisauskis\Desktop\CE...\CE ZSV PC.m 1 of 8

```
%*****
%
% PROJECT:      GAS ASSISTED INJECTION MOLDING
% PROGRAM NAME: Cross-Exp-ZSV_Model_PC
% PURPOSE:      m vs Fo for PC
% DATE:        10/25/2017
%
%*****
clear variables
% close all
clc

%% Initial Conditions

% Number of nodes
nodes = 21;

% time measurements
t = linspace(1E-10,50,1000); %sec

% time increments
dt = 0.0001;

% Temperature of mold
Tb = (115.6-32)/1.8; %degC

% Temperature of melt
Ti = (623.6-32)/1.8; %degC

% Temperature of cooling fluid
Tc = 7; %degC

%Fractional coverage asymptotic value
m_n = 0.6;

%% Physical Properties of the tube

% External Diameter
OD = 2 * (150E-3+0.00635); %m

% Internal Diameter
ID = 2 * 0.00635; %m

% External Radius
R02 = OD * 0.5; %m

% Internal Radius
R01 = ID * 0.5; %m

% Length of tube
L = 0.585; %m

% Gravitational Acceleration
g = 9.81; %m/s^2

% Diameter
```

```
D = OD; %m

% Thermal Conductivity
k_s = 16; %W/(m*K)

%Density
den_s = 7850; %kg/m^3

%Specific Heat
Cp_s = 514.5; %J/(kg*K)

% Thermal Diffusivity
a_s = k_s / (den_s * Cp_s); %m^2/s

% Step change in tube
dr2 = (R02 - R01) / (nodes - 1);

%% Physical Properties of the polymer

% Specific Heat
Cp_p = 2100; %J/(kg*K)

% Thermal Conductivity
k_p = 0.15; %W/(m*K)

% Density
den_p = 1017; %kg/m^3

% Thermal Diffusivity
a_p = k_p / (den_p * Cp_p); %m^2/s

% Temperature Sensitivity Coefficient
T_b = 1.1376E4; %K

% Zero shear viscosity
zero_visc = 1.0242*10^-4*exp(9.0679E3/(Ti+273.13));

% Frequency factor
B = zero_visc / exp(T_b / (25+273.13)); %kg/(m*s)

% Pressure gradient
delta_p = 2.42 * 10^6; %Pa

% n parameter
n = 1 - 0.03727;

% Step change in polymer
dr1 = R01 / (nodes - 1);

n_dr1 = 1.0 / (nodes - 1);

%% Physical Properties of the cooling fluid

% Specific Heat
Cp_f = 10^-6 * Tb^4 - 0.0004 * Tb^3 + 0.0488 * Tb^2 - 2.2527 * Tb + 4212; %J/(kg*K)
```

```
% Viscosity
u_f = (4 * 10^-8 * Tb^4 - 10^-5 * Tb^3 + 0.001 * Tb^2 - 0.0557 * Tb + 1.7832) * 10^-3; %kg/(m*s)

% Density
den_f = -0.0036 * Tb^2 - 0.0656 * Tb + 1000.4; %kg/m^3

% Thermal Conductivity
k_f = -9 * 10^-6 * Tb^2 + 0.0021 * Tb + 0.5607; %W/(m*K)

% Coefficient of Volume Expansion
b = (0.0036 * 2 * Tb + 0.0656) / den_f; %1/K

% Thermal Diffusivity
a_f = k_f / (den_f * Cp_f); %m^2/s

%% Heat Transfer outside the tube wall

% Initializing temperature profiles
Temp_p = zeros(nodes,2);
Temp_s = zeros(nodes,2);
Temp_fp = zeros(nodes,length(t));
Temp_fs = zeros(nodes,length(t));

% Initializing radial position profiles
for i = nodes : -1 : 1
    r_s(i) = R01 + dr2 * (i-1);
end
for i = nodes : -1 : 1
    r_p(i) = dr1 * (i-1);
end

% Boundary Condition 1: @ t=0, T_f(r,0) = Ti
Temp_p = Temp_p + Ti;

% Boundary Condition 2: @ t=0, T_s(r,0) = Tc
Temp_s = Temp_s + Tc;

% Calculation at R01
i=nodes;
j=1;

% Grashof Number
Gr = D^3 * den_f^2 * g * b * (Temp_s(i,j)-Tb) / u_f^2;

% Prandtl Number
Pr = Cp_f * u_f / k_f;

% Raleigh Number
Ra = Gr * Pr;

% Reynolds Number
Re = 1;

% Nusselt Number
Nu = 0.62*Re^(1/2)*Pr^(1/3)/[1+(0.4/Pr)^(2/3)]^(1/4);
```

```

% Heat Transfer Coefficient
h = k_f * Nu / D;

%Initialization of counter variables
count = 0;

for k = 1 : length(t)

    disp(length(t)-k+1)

    while count <= t(k)

        %counter variables incrementation
        count = count + dt;

        % Boundary Condition 3: @ r=R02, -ks*dT/dr = h(Ts(R02,t)-Tb)
        i=nodes;
        j=1;
        Temp_s(i,j+1) = Temp_s(i,j) + (2*a_s*dt/dr2^2)*(Temp_s(i-1,j)-Temp_s(i,j)-((R02+(dr2/2))/R02)*(h/k_s)*dr2*(Temp_s(i,j)-Tb));

        % Temperature profile in Solid
        for i = nodes-1 : -1 : 2

            Temp_s(i,j+1) = Temp_s(i,j) + (a_s*dt/(r_s(i)*dr2^2))*(((r_s(i)+r_s(i-1))/2)*Temp_s(i-1,j)-2*r_s(i)*Temp_s(i,j)+((r_s(i+1)+r_s(i))/2)*Temp_s(i+1,j));

        end

        % Boundary Condition 4: @ r=R01, solid-liquid interface energy balance
        i = 1;
        Temp_s(i,j+1) = Temp_s(i,j) + (2*k_s*dt*(R01+dr2/2)*(Temp_s(i+1,j)-Temp_s(i,j))/(dr2*((R01+dr2/4)*den_s*Cp_s*dr2+(R01-dr1/4)*(-0.58 * Temp_p(nodes,j) + 903)*Cp_p*dr1)))+(2*k_p*dt*((R01-dr1/2)*(Temp_p(nodes-i,j)-Temp_s(i,j))/(dr1*((R01+dr2/4)*den_s*Cp_s*dr2+(R01-dr1/4)*(-0.58 * Temp_p(nodes,j) + 903)*Cp_p*dr1)));

        Temp_p(nodes,j+1) = Temp_s(i,j+1);

        % Temperature profile in Polymer
        for i = nodes-1 : -1 : 2

            Temp_p(i,j+1) = Temp_p(i,j) + ((k_p/((-0.58 * Temp_p(i,j) + 903)*Cp_p))*dt/(r_p(i)*dr1^2))*(((r_p(i)+r_p(i-1))/2)*Temp_p(i-1,j)-2*r_p(i)*Temp_p(i,j)+((r_p(i+1)+r_p(i))/2)*Temp_p(i+1,j));

        end

        % Boundary Condition 5: @r=0, kdT/dr = 0
        i=1;
        Temp_p(i,j+1) = Temp_p(i,j) + (4*(k_p/((-0.58 * Temp_p(i,j) + 903)*Cp_p))*dt/dr1^2)*(Temp_p(i+1,j)-Temp_p(i,j));

        for i = 1 : nodes

            Temp_s(i,j) = Temp_s(i,j+1);
            Temp_p(i,j) = Temp_p(i,j+1);

```

```

        end

    end

    % Creating Temperature Profiles for Polymer and Solid
    for i = 1 : nodes

        Temp_fp(i,k) = Temp_p(i,j);
        Temp_fs(i,k) = Temp_s(i,j);

    end
end

%% Temperature Profile
figure
plot(Temp_fp(1:21,1),r_p)
hold on
plot(Temp_fs(1:21,1),r_s)
plot(Temp_fp(1:21,1),-r_p)
plot(Temp_fs(1:21,1),-r_s)
plot(Temp_fp(1:21,500),r_p)
plot(Temp_fs(1:21,500),r_s)
plot(Temp_fp(1:21,500),-r_p)
plot(Temp_fs(1:21,500),-r_s)
plot(Temp_fp(1:21,1000),r_p)
plot(Temp_fs(1:21,1000),r_s)
plot(Temp_fp(1:21,1000),-r_p)
plot(Temp_fs(1:21,1000),-r_s)

xlabel('Temperature (degC)')
ylabel('Radial Position (m)')
axis([0 350 -0.15 0.15])
% axis([0 350 -0.00635 0.00635])
% legend('t=0s','t=0s','t=25s','t=25s','t=50s','t=50s')
% legend boxon

%% Velocity Profile Calculation

%initialize velocity matrix
vel = zeros(nodes,length(t));

%velocity profile calculation
for k = 1 : length(t)

    %equation 4.33
    vel(nodes,k) = 0;

    for i = nodes-1 : -1 : 1

        f(i) = (n_dr1/2)*(delta_p/(2*L*B))^(1/n)*(((r_p(i+1)/exp(T_b/(Temp_fp(i+1,k)+273.13))))^(1/n)+((r_p(i)/exp(T_b/(Temp_fp(i,k)+273.13))))^(1/n));
        vel(i,k) = vel(i+1,k) + f(i);

    end
end

```

```

end

%normalizing the velocity profile
n_vel = zeros(nodes,length(t));

for k = 1 : length(t)

    for i = 1 : nodes

        n_vel(i,k) = vel(i,k) / vel(1,k);

    end

end

%normalizing the radial position
n_radius = zeros(2*nodes,1);

for i = 1 : nodes
    n_radius(i) = r_p(i) / R01;
end

for i = 1 : nodes
    n_radius(i+nodes) = r_s(i) / R01;
end

%% plot velocity profile
figure
hold on
plot(n_vel(1:21,1),n_radius(1:21,1))
plot(n_vel(1:21,20),n_radius(1:21,1))
plot(n_vel(1:21,40),n_radius(1:21,1))
plot(n_vel(1:21,60),n_radius(1:21,1))
plot(n_vel(1:21,80),n_radius(1:21,1))
plot(n_vel(1:21,100),n_radius(1:21,1))
plot(n_vel(1:21,1),-n_radius(1:21,1))
plot(n_vel(1:21,20),-n_radius(1:21,1))
plot(n_vel(1:21,40),-n_radius(1:21,1))
plot(n_vel(1:21,60),-n_radius(1:21,1))
plot(n_vel(1:21,80),-n_radius(1:21,1))
plot(n_vel(1:21,100),-n_radius(1:21,1))
ylabel('Normalized Radius')
xlabel('Normalized Velocity')
legend('t = 1 s', 't = 20 s', 't = 40 s', 't = 60 s', 't = 80 s', 't = 100 s')
legend boxon
box on

%% Fractional Coverage Calculation

%initialize vectors
m = zeros(length(t),1);
Rx = zeros(length(t),1);

for k = 1 : length(t)

    %initializing summation variables

```

```

sum = 0;

for i = 1 : nodes-1

    %integrate through ru*
    sum = sum + (n_dr1 / 2) * (n_radius(i)*n_vel(i,k) + n_radius(i+1)*n_vel(i+1,k));

end

%integrate through ru*
Rx(k) = R01 * sqrt(4 * sum);

%Fractional coverage when integrating through ru*
m(k) = 1 - (1 - m_n) * (Rx(k) / R01)^2-0.033;

end

%% Experimental Data

Fo_exp = [0.024058062 0.023212858 0.022752786 0.022245284 0.021805132 0.021334626 0.020703806
0.020302547 0.020025555 0.0188417 0.018269352 0.017706224 0.017191133 0.016183945 0.01573408
0.015466612 0.012500676 0.009558113 0.00661555 0.003672987 0.000730423 0 0.020624882
0.019661766 0.019155687 0.018606921 0.018137173 0.017640106 0.016980353 0.016564012 0.016277914
0.015378639 0.015065221 0.014483857 0.013914472 0.013395682 0.012386157 0.011937111 0.011670626
0.009928067 0.007668498 0.005408928 0.003149358 0.017492219 0.016570842 0.016083735 0.015553941
0.015099371 0.014617481 0.0139767 0.013571742 0.013293233 0.012416725 0.011543191 0.0104793
0.009491024 0.009051085 0.008789912 0.007908209 0.020522433 0.019524883 0.019010741 0.017989666
0.017496393 0.016845652 0.016436994 0.016156967 0.015280459 0.014976147 0.014413088 0.013863248
0.013363524 0.011964147 0.011709269 0.009561907 0.007387712 0.021607254 0.020811092 0.020380901
0.019908023 0.019498986 0.019062629 0.018478765 0.018107956 0.017852213 0.017044005 0.016760942
0.016234217 0.015716436 0.015243179 0.014318649 0.01390603 0.013660793 0.010888053 0.008366669
0.005845285 0.003323901 0.000802517 0.020658273 0.019731678 0.019280144 0.018809637 0.018419572
0.018017365 0.017497531 0.017176524 0.016958725 0.016287115 0.016057174 0.015635631 0.015228475
0.014861936 0.014159454 0.013851105 0.013669217 0.01130354 0.008685399 0.003449116 0.000830975
0.021628882 0.02085359 0.020432885 0.019969493 0.019568045 0.019139276 0.018564898 0.018199781
0.017947832 0.017151007 0.016351844 0.015840513 0.015372947 0.014459042 0.013808396 0.02165772
0.021021208 0.020655996 0.020243354 0.019879091 0.019484473 0.018948513 0.01860417 0.018365122
0.017602447 0.017333044 0.016828994 0.016330371 0.015872197 0.014971253 0.01456692 0.011614682
0.008973774 0.003691959 0.001051051 0 0.02060591 0.019497753 0.018948513 0.018370814
0.017888165 0.017387304 0.016735614 0.016330941 0.016055467 0.015201725 0.014908038 0.014367927
0.013844276 0.013371303 0.012059955 0.011822979 0.009816132 0.00765332 0.005490508 0.003327695
0.001164883];

m_exp = [0.697294689 0.724263224 0.723513367 0.735492284 0.723607207 0.724449991 0.727618456
0.717437048 0.726801285 0.725024622 0.720672799 0.728904796 0.720264006 0.722674305 0.718447447
0.718957268 0.70168097 0.66145734 0.648457964 0.647927231 0.624885193 0.596755298 0.670739472
0.703137029 0.691550379 0.698364593 0.701241289 0.702084104 0.702734839 0.695427813 0.700071162
0.69118994 0.697962535 0.696915367 0.695680165 0.697149418 0.685426476 0.697779619 0.691462878
0.652087133 0.645164384 0.62876281 0.619778545 0.65467158 0.665676326 0.665864314 0.663917007
0.665206569 0.66760097 0.664453562 0.659576701 0.653769429 0.648133579 0.655552792 0.651622842
0.648423281 0.648282304 0.633809094 0.628643675 0.696792714 0.704865477 0.707882141 0.708855373
0.705523156 0.699458032 0.701641672 0.704577476 0.699915757 0.702106235 0.702440258 0.68981505
0.70152703 0.676931134 0.637488469 0.623183955 0.613461083 0.68239709 0.69807312 0.707196133
0.706231596 0.707991626 0.70932566 0.708581543 0.703773498 0.708160712 0.690731214 0.703222289
0.703554796 0.702598996 0.702197434 0.692279374 0.697874714 0.697550677 0.660354829 0.656144062
0.640846623 0.634127931 0.633314738 0.670580778 0.701208446 0.702841462 0.701430498 0.707964274
0.704267557 0.704437699 0.697208985 0.700352376 0.689056354 0.698702575 0.701758078 0.698799327];

```

```
0.699270899 0.686868841 0.695377846 0.692602561 0.628149629 0.622848174 0.61980335 0.616109316✓  
0.672382415 0.699464312 0.701181268 0.701738315 0.703221545 0.705989659 0.701207358 0.700674887✓  
0.703816372 0.698529973 0.706125555 0.704326992 0.708604945 0.682532306 0.673952105 0.70186614✓  
0.704966546 0.715469093 0.709629477 0.711614645 0.705424738 0.704353297 0.710230389 0.708774738✓  
0.703280722 0.711482142 0.712119017 0.714311394 0.710045123 0.705971812 0.703328536 0.699779951✓  
0.707333813 0.662368545 0.641571248 0.637470269 0.682723977 0.704219021 0.699624337 0.701748647✓  
0.703458037 0.70243286 0.704290352 0.697302691 0.698056443 0.696005632 0.694936722 0.698370737✓  
0.698107561 0.69501785 0.682227046 0.681293654 0.641569264 0.613927755 0.610523635 0.610874927✓  
0.60715379];
```

```
%% Fourier Number
```

```
Fo = a_p*t/R01^2;
```

```
%% Plot m vs Fo
```

```
figure  
semilogx(Fo, m, Fo_exp, m_exp, 'rs')  
xlabel('Fo')  
ylabel('m')  
% title('Polycarbonate Model')  
legend('Simulated Data', 'Experimental Data')  
axis([1E-4 1E-1 0.55 0.95])  
box on  
legend boxoff  
legend('Location', 'NorthWest')
```

```
*****  
%  
% CODE END  
%  
*****
```


10 Appendix C: MATLAB code for PC Simulation Comparison

11/1/17 7:53 AM C:\Users\estanisauskis\Des...\Power Model PC.m 1 of 11

```
*****
%
% PROJECT:      GAS ASSISTED INJECTION MOLDING
% PROGRAM NAME: CE_ZSV_PC_Sim_vs_Exp.m
% PURPOSE:      m vs Fo for PC
% DATE:         10/26/2017
%
*****
clear variables
close all
clc

%% Load Excel Files

% delay time data references
% data_points = [1 23 44 9 31 52 11 34 54 21];
% Ti_dp = [346.35 348.25 351.15 347.65 349.05 351.75 346.82 348.645 350.942 347.65]; %degC
% gas pressure data references
data_points = [156 23 135 164 31 143 167 34 146 176 154];
Ti_dp = [348.27 347.25 347.54 349.21 349.05 348 348.721 348.645 347.362 348.85 348.83]; %degC
% shot size data references
% data_points = [120 23 99 128 31 107 130 34 110 119 ];
% Ti_dp = [347.55 348.25 348.25 348.15 349.05 348.65 348.69 348.645 347.64 347.45]; %degC
% piston speed data references
% data_points = [60 23 77 67 31 85 70 34 88 98];
% Ti_dp = [348.35 348.25 347.55 349.15 349.05 348.15 347.585 348.645 347.955 347.55]; %degC

for data_point_no = 1:length(data_points)
% find files
convertedrestime = xlsread('Fractional_Coverage_Comparison.xlsx','G2:G177');
convertedrowlength = xlsread('Fractional_Coverage_Comparison.xlsx','K2:K177');
factorr = xlsread('Fractional_Coverage_Comparison.xlsx','J2:J177');

dp = data_points(data_point_no);

converted_res_time=zeros(length(data_points),1);
converted_rowlength=zeros(length(data_points),1);
factor=zeros(length(data_points),1);

for i = 1:length(data_points)
    dp = data_points(i);
    converted_res_time(i) = convertedrestime(dp);
    converted_rowlength(i) = convertedrowlength(dp);
    factor(i) = factorr(dp);
end

%% Initial Conditions

% Number of nodes
nodes = 21;

% time measurements
%t = [5 10 20 30 40 50 60 90 120 180 240 480]; %sec
t = linspace(1E-10,50,1000); %sec

% time increments
```

```
dt = 0.0001;

% Temperature of mold
Tb = (115.6-32)/1.8; %degC

% Temperature of cooling fluid
Tc = 7; %degC

% Temperature of melt
Ti = Ti_dp(data_point_no); %degC

%Fractional coverage asymptotic value
m_n = 0.6;

%% Physical Properties of the tube

% External Diameter
OD = 2 * (150E-3+0.00635); %m

% Internal Diameter
ID = 2 * 0.00635; %m

% External Radius
R02 = OD * 0.5; %m

% Internal Radius
R01 = ID * 0.5; %m

% Length of tube
L = 0.585; %m

% Gravitational Acceleration
g = 9.81; %m/s^2

% Diameter
D = OD; %m

% Thermal Conductivity
k_s = 16; %W(m*K)

%Density
den_s = 7850; %kg/m^3

%Specific Heat
Cp_s = 514.5; %J/(kg*K)

% Thermal Diffusivity
a_s = k_s / (den_s * Cp_s); %m^2/s

% Step change in tube
dr2 = (R02 - R01) / (nodes - 1);

%% Physical Properties of the polymer

% Specific Heat
Cp_p = 2100; %J/(kg*K)
```

```
% Thermal Conductivity
k_p = 0.15; %W/(m*K)

% Density
den_p = 1017; %kg/m^3

% Thermal Diffusivity
a_p = k_p / (den_p * Cp_p); %m^2/s

% Flow Activation Energy
delta_H_R = 1.1376E4; %K

% Zero shear viscosity
zero_visc = 1.0242*10^-4*exp(9.0679E3/(Ti+273.13));

% Frequency factor
A = zero_visc / exp(delta_H_R / (25+273.13)); %kg/(m*s)

% Pressure gradient
delta_p = 2.42 * 10^6; %Pa

% n parameter
n = 1 - 0.03727;

% Step change in polymer
dr1 = R01 / (nodes - 1);

n_dr1 = 1.0 / (nodes - 1);

%% Physical Properties of the cooling fluid

% Specific Heat
Cp_f = 10^-6 * Tb^4 - 0.0004 * Tb^3 + 0.0488 * Tb^2 - 2.2527 * Tb + 4212; %J/(kg*K)

% Viscosity
u_f = (4 * 10^-8 * Tb^4 - 10^-5 * Tb^3 + 0.001 * Tb^2 - 0.0557 * Tb + 1.7832) * 10^-3; %kg/(m*s)

% Density
den_f = -0.0036 * Tb^2 - 0.0656 * Tb + 1000.4; %kg/m^3

% Thermal Conductivity
k_f = -9 * 10^-6 * Tb^2 + 0.0021 * Tb + 0.5607; %W/(m*K)

% Coefficient of Volume Expansion
b = (0.0036 * 2 * Tb + 0.0656) / den_f; %1/K

% Thermal Diffusivity
a_f = k_f / (den_f * Cp_f); %m^2/s

%% Heat Transfer outside the tube wall

% Initializing temperature profiles
Temp_p = zeros(nodes,2);
Temp_s = zeros(nodes,2);
Temp_fp = zeros(nodes,length(t));
```

```

Temp_fs = zeros(nodes,length(t));

% Initializing radial position profiles
for i = nodes : -1 : 1
    r_s(i) = R01 + dr2 * (i-1);
end
for i = nodes : -1 : 1
    r_p(i) = dr1 * (i-1);
end

% Boundary Condition 1: @ t=0, T_f(r,0) = Ti
Temp_p = Temp_p + Ti;

% Boundary Condition 2: @ t=0, T_s(r,0) = Ti
Temp_s = Temp_s + Tc;

% Calculation at R01
i=nodes;
j=1;

% Grashof Number
Gr = D^3 * den_f^2 * g * b * (Temp_s(i,j)-Tb) / u_f^2;

% Prandtl Number
Pr = Cp_f * u_f / k_f;

% Raleigh Number
Ra = Gr * Pr;

% Reynolds Number
Re = 1;

% Nusselt Number
Nu = 0.62*Re^(1/2)*Pr^(1/3)/[1+(0.4/Pr)^(2/3)]^(1/4);

% Heat Transfer Coefficient
h = k_f * Nu / D;

%Initialization of counter variables
count = 0;

for k = 1 : length(t)

    disp(length(t)-k+1)

    while count <= t(k)

        %counter variables incrementation
        count = count + dt;

        % Boundary Condition 3: @ r=R02, -ks*dT/dr = h(Ts(R02,t)-Tb)
        i=nodes;
        j=1;
        Temp_s(i,j+1) = Temp_s(i,j) + (2*a_s*dt/dr2^2)*(Temp_s(i-1,j)-Temp_s(i,j)-((R02+(dr2/2))
        /R02)*(h/k_s)*dr2*(Temp_s(i,j)-Tb));
    end
end

```

```

% Temperature profile in Solid
for i = nodes-1 : -1 : 2

    Temp_s(i,j+1) = Temp_s(i,j) + (a_s*dt/(r_s(i)*dr2^2))*((r_s(i)+r_s(i-1))/2)*Temp_s(
(i-1,j))-2*r_s(i)*Temp_s(i,j)+((r_s(i+1)+r_s(i))/2)*Temp_s(i+1,j));

end

% Boundary Condition 4: @ r=R01, solid-liquid interface energy balance
i = 1;
Temp_s(i,j+1) = Temp_s(i,j) + (2*k_s*dt*(R01+dr2/2)*(Temp_s(i+1,j)-Temp_s(i,j))/(dr2*
((R01+dr2/4)*den_s*Cp_s*dr2+(R01-dr1/4)*(-0.58 * Temp_p(nodes,j) + 903)*Cp_p*dr1)))+(2*k_p*dt*
(R01-dr1/2)*(Temp_p(nodes-i,j)-Temp_s(i,j))/(dr1*((R01+dr2/4)*den_s*Cp_s*dr2+(R01-dr1/4)*(-0.58 *
Temp_p(nodes,j) + 903)*Cp_p*dr1)));

Temp_p(nodes,j+1) = Temp_s(i,j+1);

% Temperature profile in Polymer
for i = nodes-1 : -1 : 2

    Temp_p(i,j+1) = Temp_p(i,j) + ((k_p/((-0.58 * Temp_p(i,j) + 903)*Cp_p))*dt/(r_p(i)
*dr1^2))*((r_p(i)+r_p(i-1))/2)*Temp_p(i-1,j)-2*r_p(i)*Temp_p(i,j)+((r_p(i+1)+r_p(i))/2)*Temp_p
(i+1,j));

end

% Boundary Condition 5: @r=0, kdT/dr = 0
i=1;
Temp_p(i,j+1) = Temp_p(i,j) + (4*(k_p/((-0.58 * Temp_p(i,j) + 903)*Cp_p))*dt/dr1^2)*
(Temp_p(i+1,j)-Temp_p(i,j));

for i = 1 : nodes

    Temp_s(i,j) = Temp_s(i,j+1);
    Temp_p(i,j) = Temp_p(i,j+1);

end

end

% Creating Temperature Profiles for Polymer and Solid
for i = 1 : nodes

    Temp_fp(i,k) = Temp_p(i,j);
    Temp_fs(i,k) = Temp_s(i,j);

end

end

%% Velocity Profile Calculation

%initialize velocity matrix
vel = zeros(nodes,length(t));

%velocity profile calculation
for k = 1 : length(t)

```

```
vel(nodes,k) = 0;

for i = nodes-1 : -1 : 1

    f(i) = (n_dr1/2)*(delta_p/(2*L*A))^(1/n)*(((r_p(i+1)/exp(delta_H_R/(Temp_fp(i+1,k)+273.13)))^(1/n)+((r_p(i)/exp(delta_H_R/(Temp_fp(i,k)+273.13)))^(1/n)));
    vel(i,k) = vel(i+1,k) + f(i);

end

end

%normalizing the velocity profile
n_vel = zeros(nodes,length(t));

for k = 1 : length(t)

    for i = 1 : nodes

        n_vel(i,k) = vel(i,k) / vel(1,k);

    end

end

%normalizing the radial position
n_radius = zeros(2*nodes,1);

for i = 1 : nodes
    n_radius(i) = r_p(i) / R01;
end

for i = 1 : nodes
    n_radius(i+nodes) = r_s(i) / R01;
end

% plot velocity profile
figure(1)
hold on
plot(n_vel(1:21,1),n_radius(1:21,1))
plot(n_vel(1:21,20),n_radius(1:21,1))
plot(n_vel(1:21,40),n_radius(1:21,1))
plot(n_vel(1:21,60),n_radius(1:21,1))
plot(n_vel(1:21,80),n_radius(1:21,1))
plot(n_vel(1:21,100),n_radius(1:21,1))
ylabel('Normalized Radius')
xlabel('Normalized Velocity')

%% Fractional Coverage Calculation

%initialize vectors
m = zeros(length(t),1);
Rx = zeros(length(t),1);

for k = 1 : length(t)
```

```

%initializing summation variables
sum = 0;

for i = 1 : nodes-1

    %integrate through ru*
    sum = sum + (n_dr1 / 2) * (n_radius(i)*n_vel(i,k) + n_radius(i+1)*n_vel(i+1,k));

end

%integrate through ru*
Rx(k) = R01 * sqrt(4 * sum);

%Fractional coverage when integrating through ru*
m(k) = 1 - (1 - m_n) * (Rx(k) / R01)^2-0.03;

end

%% Experimental Data

Fo_exp = [0.024058062 0.023212858 0.022752786 0.022245284 0.021805132 0.021334626 0.020703806 ✓
0.020302547 0.020025555 0.0188417 0.018269352 0.017706224 0.017191133 0.016183945 0.01573408 ✓
0.015466612 0.012500676 0.009558113 0.00661555 0.003672987 0.000730423 0 0.020624882 ✓
0.019661766 0.019155687 0.018606921 0.018137173 0.017640106 0.016980353 0.016564012 0.016277914 ✓
0.015378639 0.015065221 0.014483857 0.013914472 0.013395682 0.012386157 0.011937111 0.011670626 ✓
0.009928067 0.007668498 0.005408928 0.003149358 0.017492219 0.016570842 0.016083735 0.015553941 ✓
0.015099371 0.014617481 0.0139767 0.013571742 0.013293233 0.012416725 0.011543191 0.0104793 ✓
0.009491024 0.009051085 0.008789912 0.007908209 0.020522433 0.019524883 0.019010741 0.017989666 ✓
0.017496393 0.016845652 0.016436994 0.016156967 0.015280459 0.014976147 0.014413088 0.013863248 ✓
0.013363524 0.011964147 0.011709269 0.009561907 0.007387712 0.021607254 0.020811092 0.020380901 ✓
0.019908023 0.019498986 0.019062629 0.018478765 0.018107956 0.017852213 0.017044005 0.016760942 ✓
0.016234217 0.015716436 0.015243179 0.014318649 0.01390603 0.013660793 0.010888053 0.008366669 ✓
0.005845285 0.003323901 0.000802517 0.020658273 0.019731678 0.019280144 0.018809637 0.018419572 ✓
0.018017365 0.017497531 0.017176524 0.016958725 0.016287115 0.016057174 0.015635631 0.015228475 ✓
0.014861936 0.014159454 0.013851105 0.013669217 0.01130354 0.008685399 0.003449116 0.000830975 ✓
0.021628882 0.02085359 0.020432885 0.019969493 0.019568045 0.019139276 0.018564898 0.018199781 ✓
0.017947832 0.017151007 0.016351844 0.015840513 0.015372947 0.014459042 0.013808396 0.02165772 ✓
0.021021208 0.020655996 0.020243354 0.019879091 0.019484473 0.018948513 0.01860417 0.018365122 ✓
0.017602447 0.017333044 0.016828994 0.016330371 0.015872197 0.014971253 0.01456692 0.011614682 ✓
0.008973774 0.003691959 0.001051051 0 0.02060591 0.019497753 0.018948513 0.018370814 ✓
0.017888165 0.017387304 0.016735614 0.016330941 0.016055467 0.015201725 0.014908038 0.014367927 ✓
0.013844276 0.013371303 0.012059955 0.011822979 0.009816132 0.00765332 0.005490508 0.003327695 ✓
0.001164883];

m_exp = [0.697294689 0.724263224 0.723513367 0.735492284 0.723607207 0.724449991 0.727618456 ✓
0.717437048 0.726801285 0.725024622 0.720672799 0.728904796 0.720264006 0.722674305 0.718447447 ✓
0.718957268 0.70168097 0.66145734 0.648457964 0.647927231 0.624885193 0.596755298 0.670739472 ✓
0.703137029 0.691550379 0.698364593 0.701241289 0.702084104 0.702734839 0.695427813 0.700071162 ✓
0.69118994 0.697962535 0.696915367 0.695680165 0.697149418 0.685426476 0.697779619 0.691462878 ✓
0.652087133 0.645164384 0.62876281 0.619778545 0.65467158 0.665676326 0.665864314 0.663917007 ✓
0.665206569 0.66760097 0.664453562 0.659576701 0.653769429 0.648133579 0.655552792 0.651622842 ✓
0.648423281 0.648282304 0.633809094 0.628643675 0.696792714 0.704865477 0.707882141 0.708855373 ✓
0.705523156 0.699458032 0.701641672 0.704577476 0.699915757 0.702106235 0.702440258 0.68981505 ✓
0.70152703 0.676931134 0.637488469 0.623183955 0.613461083 0.68239709 0.69807312 0.707196133 ✓
0.706231596 0.707991626 0.70932566 0.708581543 0.703773498 0.708160712 0.690731214 0.703222289 ✓

```

```

0.703554796 0.702598996 0.702197434 0.692279374 0.697874714 0.697550677 0.660354829 0.656144062 ✓
0.640846623 0.634127931 0.633314738 0.670580778 0.701208446 0.702841462 0.701430498 0.707964274 ✓
0.704267557 0.704437699 0.697208985 0.700352376 0.689056354 0.698702575 0.701758078 0.698799327 ✓
0.699270899 0.686868841 0.695377846 0.692602561 0.628149629 0.622848174 0.61980335 0.616109316 ✓
0.672382415 0.699464312 0.701181268 0.701738315 0.703221545 0.705989659 0.701207358 0.700674887 ✓
0.703816372 0.698529973 0.706125555 0.704326992 0.708604945 0.682532306 0.673952105 0.70186614 ✓
0.704966546 0.715469093 0.709629477 0.711614645 0.705424738 0.704353297 0.710230389 0.708774738 ✓
0.703280722 0.711482142 0.712119017 0.714311394 0.710045123 0.705971812 0.703328536 0.699779951 ✓
0.707333813 0.662368545 0.641571248 0.637470269 0.682723977 0.704219021 0.699624337 0.701748647 ✓
0.703458037 0.70243286 0.704290352 0.697302691 0.698056443 0.696005632 0.694936722 0.698370737 ✓
0.698107561 0.69501785 0.682227046 0.681293654 0.641569264 0.613927755 0.610523635 0.610874927 ✓
0.60715379];

```

```
%% Fourier Number
```

```
Fo = a_p*t/R01^2;
```

```
%% Plot m vs Fo
```

```

figure (2)
semilogx(Fo, m, Fo_exp, m_exp, 'rs')
xlabel('Fo')
ylabel('m')
axis([1E-5 1E-1 0.55 0.95])
title('Polycarbonate Model')
legend('Simulated Data', 'Experimental Data')

```

```
%% Determination of shear_rate
```

```
% turn converted_res_time and converted_rowlength into integers
```

```

for i = 1:length(converted_rowlength)
    converted_rowlength(i) = round(converted_rowlength(i));
end
for i = 1:length(converted_res_time)
    converted_res_time(i) = round(converted_res_time(i));
end

```

```
% calculate n_vel_sim
```

```

n_vel_sim = zeros(length(converted_rowlength),length(converted_res_time));
for i = 1:length(converted_rowlength)
    if converted_rowlength(i) < 101
        if converted_res_time(i) < 101
            n_vel_sim(i,i) = n_vel(converted_rowlength(i),converted_res_time(i));
        else
            n_vel_sim(i,i) = 0;
        end
    else
        n_vel_sim(i,k) = 0;
    end
end

```

```
% non_normalize n_vel_sim
```

```

vel_sim = zeros(length(converted_rowlength),1);
for i = 1:length(converted_rowlength)
    vel_sim(i,1) = n_vel_sim(i,i)*factor(i);
end

```



```

% determine m_sim
m_sim = zeros(length(converted_res_time),1);
for i = 1:length(converted_res_time)
    if converted_res_time(i) < 1001
        m_sim(i) = m(converted_res_time(i));
    else
        m_sim(i) = 0;
    end
end

% calculate shear rate (gamma_dot)
vel_diff = zeros(length(vel_sim),1);
t_diff = zeros(length(vel_sim),1);
n_gamma_dot = zeros(length(vel_sim),1);
gamma_dot = zeros(length(vel_sim),1);
for i =1:length(vel_sim)
    if converted_res_time(i) < 1001
        vel_diff(i) = n_vel(converted_rowlength(i)-1,converted_res_time(i))-n_vel
(converted_rowlength(i),converted_res_time(i));
        t_diff(i) = 0.05;
        n_gamma_dot(i) = vel_diff(i)/t_diff(i);
        gamma_dot(i) = n_gamma_dot(i)*factor(i);
    else
        gamma_dot(i)=0;
    end
end

%% Plot Viscosity vs shear rate with the newly acquired shear rate values
% Method 1: Cross-exponential Model
lenn = 1E2;
Ti = Ti+273.15; %K
m_power = A.*exp(delta_H_R./Ti); %Pa-s
shear_rate = logspace(-3, 10, lenn);
n_p = 0.03727;
B = 1.0242E-4;
Tb = 9.0679E3;
no = B.*exp(Tb./Ti);
tau_star = 9.4221E5;
for k = 1:length(shear_rate)
    for i = 1:length(Ti)
        viscosity(i,k) = no(i)/(1+(no(i)*shear_rate(k)/tau_star)^(1-n_p));
        visc_dp(data_point_no,k) = viscosity(i,k);
    end
end
figure (3)
loglog(shear_rate, viscosity(1,1:lenn))
hold on
% loglog(shear_rate, viscosity(2,1:lenn))
% loglog(shear_rate, viscosity(3,1:lenn))
% loglog(shear_rate, viscosity(4,1:lenn))
% loglog(shear_rate, viscosity(5,1:lenn))
% loglog(shear_rate, viscosity(6,1:lenn))
% loglog(shear_rate, viscosity(7,1:lenn))
% loglog(shear_rate, viscosity(8,1:lenn))
% loglog(shear_rate, viscosity(9,1:lenn))

```

```

% loglog(shear_rate, viscosity(10,1:lenn))

% plot data points
n_p = 0.03727;
B = 1.0242E-4;
Tb = 9.0679E3;
no_exp = B*exp(Tb/Ti);
tau_star = 9.4221E5;
for i = 1:length(gamma_dot)
    visc_exp(i) = no_exp/(1+(no_exp*gamma_dot(i)/tau_star)^(1-n_p));
end
visc_exp_dp(data_point_no) = visc_exp(1);
loglog(gamma_dot(1), visc_exp(1), 'rs')
leg_ent = ['ABCDEFGHJKLMN'];
legend(leg_ent(data_point_no))
% loglog(gamma_dot(2), visc_exp(2), 'bo')
% loglog(gamma_dot(3), visc_exp(3), 'gs')
% legend('high delay time', 'base delay time', 'low delay time')
xlabel('\gamma_{dot} [1/s]')
ylabel('\eta [Pa-s]')
title('CE Model: PC')
hold off

% CONCLUSION: All shear-rates tested are before elbow region. Therefore,
% assumption that fluid behavior is Newtonian is valid for all points

%% Plot m_sim vs m_exp
figure (4)
m_exp = [m_exp(data_points(data_point_no))];
m_exp_dp(data_point_no) = m_exp;
m_sim_dp(data_point_no) = m_sim(data_point_no);
plot(m_sim(1), m_exp(1), 'rs')
xlabel('m_{sim}')
ylabel('m_{exp}')
leg_ent = ['ABCDEFGHJKLMN'];
legend(leg_ent(data_point_no))
title('PC- Rowlength: 90mm')
axis([0.5 1 0.5 1])
hold on
line = linspace(0.5,1,100);
plot(line,line)

end

%% PLOT RESULTS
figure(100)
loglog(shear_rate, visc_dp(1,1:lenn))
marker_point = ['s', '>', 'x', 's', '>', 'x', 's', '>', 'x', 's', '>', 'x'];
hold on
for i = 2:length(visc_exp_dp)
    loglog(shear_rate, visc_dp(i,1:lenn))
end
for i = 1:length(visc_exp_dp)
    loglog(gamma_dot(i), visc_exp_dp(i), marker_point(i))
end
title('Varying Pre-charge Gas Pressure Setting')

```

```

legend('High 90mm','Base 90mm','Low 90mm','High 285mm','Base 285mm','Low 285mm','High 343mm','Base 343mm','Low 343mm','High 505mm','Low 505mm','Perfect Simulation')
legend boxoff
box on
legend('Location','SouthWest')
set(legend,'FontSize',7)
xlabel('\gamma_{dot} [1/s]')
ylabel('\eta [Pa-s]')
hold off

figure(200)
hold on
for i = 1:length(m_sim_dp)
    plot(m_sim_dp(i), m_exp_dp(i),marker_point(i))
end
line = linspace(0.5,1,100);
plot(line,line)
xlabel('m_{sim}')
ylabel('m_{exp}')
legend('High 90mm','Base 90mm','Low 90mm','High 285mm','Base 285mm','Low 285mm','High 343mm','Base 343mm','Low 343mm','High 505mm','Low 505mm','Perfect Simulation');
title('Varying Pre-charge Gas Pressure Setting')
axis([0.5 1 0.5 1])
box on
legend boxoff
legend('Location','NorthWest')

%*****
%
%
%
%
%
%
%*****
CODE END
%*****

```

11 Appendix D: MATLAB code for PS: Cross-Exponential Model

11/1/17 8:27 AM C:\Users\estanisauskis\Desktop\...\CE Model PS.m 1 of 9

```
*****
%
% PROJECT:          GAS ASSISTED INJECTION MOLDING
% PROGRAM NAME: Kaminski_Implementation_of_Cross-exponential_Model
% PURPOSE:          m vs Fo for PS
% DATE:             10/27/2017
% DESCRIPTION: This code implements the Cross-exponential model and
% solves the differential equation obtained for the velocity profile with
% the use of the shooting method.
%
*****
clear all
clc
%% Initial Conditions

% Number of nodes
nodes = 157; %this number is changed from 21 to 157 because the Mathematica solution gives 157 data points

% time measurements
t = linspace(1E-10,50,1000); %sec

% time increments
dt = 0.0001;

% Temperature of mold
Tb = (180-32)/1.8; %degC

% Temperature of melt
Ti = (485.6-32)/1.8; %degC

% Temperature of cooling water
Tc = 25; %degC

%Fractional coverage asymptotic value
n_p = 0.2968; %n parameter for Cross model
if n_p >= 0.3
    m_n = 0.1516*log(n_p)+0.7259; %formulas obtained from Poslinski & Coyle
else
    m_n = -0.1428*n_p^2+0.2626*n_p+0.4782;
end

%% Physical Properties of the tube

% External Diameter
OD = 2 * (150E-3+0.00635); %m

% Internal Diameter
ID = 2 * 0.00635; %m

% External Radius
R02 = OD * 0.5; %m

% Internal Radius
R01 = ID * 0.5; %m
```

```
% Length of tube
L = 0.585; %m

% Gravitational Acceleration
g = 9.81; %m/s^2

% Diameter
D = OD; %m

% Thermal Conductivity
k_s = 16; %W/(m*K)

%Density
den_s = 7850; %kg/m^3

%Specific Heat
Cp_s = 514.5; %J/(kg*K)

% Thermal Diffusivity
a_s = k_s / (den_s * Cp_s); %m^2/s

% Step change in tube
dr2 = (R02 - R01) / (nodes - 1);

%% Physical Properties of the polymer

% Specific Heat
Cp_p = 2100; %J/(kg*K)

% Thermal Conductivity
k_p = 0.15; %W/(m*K)

% Density
den_p = 940; %kg/m^3

% Thermal Diffusivity
a_p = k_p / (den_p * Cp_p); %m^2/s

% Flow Activation Energy
delta_H_R = 9.0679E3; %K

% Zero shear viscosity
zero_visc = 1.6783*10^-6*exp(1.1376*10^4/(Ti+273.13));

% Frequency factor
A = zero_visc / exp(delta_H_R / (25+273.13)); %kg/(m*s)

% Pressure gradient
delta_p = 2.42 * 10^6; %Pa

% Step change in polymer
dr1 = R01 / (nodes - 1);

n_dr1 = 1.0 / (nodes - 1);

%% Physical Properties of the cooling fluid
```

```
% Specific Heat
Cp_f = 10^-6 * Tb^4 - 0.0004 * Tb^3 + 0.0488 * Tb^2 - 2.2527 * Tb + 4212; %J/(kg*K)

% Viscosity
u_f = (4 * 10^-8 * Tb^4 - 10^-5 * Tb^3 + 0.001 * Tb^2 - 0.0557 * Tb + 1.7832) * 10^-3; %kg/(m*s)

% Density
den_f = -0.0036 * Tb^2 - 0.0656 * Tb + 1000.4; %kg/m^3

% Thermal Conductivity
k_f = -9 * 10^-6 * Tb^2 + 0.0021 * Tb + 0.5607; %W/(m*K)

% Coefficient of Volume Expansion
b = (0.0036 * 2 * Tb + 0.0656) / den_f; %1/K

% Thermal Diffusivity
a_f = k_f / (den_f * Cp_f); %m^2/s

%% Heat Transfer outside the tube wall

% Initializing temperature profiles
Temp_p = zeros(nodes,2);
Temp_s = zeros(nodes,2);
Temp_fp = zeros(nodes,length(t));
Temp_fs = zeros(nodes,length(t));

% Initializing radial position profiles
for i = nodes : -1 : 1
    r_s(i) = R01 + dr2 * (i-1);
end
for i = nodes : -1 : 1
    r_p(i) = dr1 * (i-1);
end

% Boundary Condition 1: @ t=0, T_f(r,0) = Ti
Temp_p = Temp_p + Ti;

% Boundary Condition 2: @ t=0, T_s(r,0) = Tc
Temp_s = Temp_s + Tc;

% Calculation at R01
i=nodes;
j=1;

% Grashof Number
Gr = D^3 * den_f^2 * g * b * (Temp_s(i,j)-Tb) / u_f^2;

% Prandtl Number
Pr = Cp_f * u_f / k_f;

% Raleigh Number
Ra = Gr * Pr;

% Reynolds Number
Re = 1;
```

```

% Nusselt Number
Nu = 0.62*Re^(1/2)*Pr^(1/3)/[1+(0.4/Pr)^(2/3)]^(1/4);

% Heat Transfer Coefficient
h = k_f * Nu / D;

%Initialization of counter variables
count = 0;

for k = 1 : length(t)

    disp(length(t)-k+1)

    while count <= t(k)

        %counter variables incrementation
        count = count + dt;

        % Boundary Condition 3: @ r=R02, -ks*dT/dr = h(Ts(R02,t)-Tb)
        i=nodes;
        j=1;
        Temp_s(i,j+1) = Temp_s(i,j) + (2*a_s*dt/dr2^2)*(Temp_s(i-1,j)-Temp_s(i,j)-((R02+(dr2/2))/R02)*(h/k_s)*dr2*(Temp_s(i,j)-Tb));

        % Temperature profile in Solid
        for i = nodes-1 : -1 : 2

            Temp_s(i,j+1) = Temp_s(i,j) + (a_s*dt/(r_s(i)*dr2^2))*(((r_s(i)+r_s(i-1))/2)*Temp_s(i-1,j)-2*r_s(i)*Temp_s(i,j)+((r_s(i+1)+r_s(i))/2)*Temp_s(i+1,j));

        end

        % Boundary Condition 4: @ r=R01, solid-liquid interface energy balance
        i = 1;
        Temp_s(i,j+1) = Temp_s(i,j) + (2*k_s*dt*(R01+dr2/2)*(Temp_s(i+1,j)-Temp_s(i,j))/(dr2*((R01+dr2/4)*den_s*Cp_s*dr2+(R01-dr1/4)*(-0.58 * Temp_p(nodes,j) + 903)*Cp_p*dr1)))+(2*k_p*dt*((R01-dr1/2)*(Temp_p(nodes,j)-Temp_s(i,j))/(dr1*((R01+dr2/4)*den_s*Cp_s*dr2+(R01-dr1/4)*(-0.58 * Temp_p(nodes,j) + 903)*Cp_p*dr1)));

        Temp_p(nodes,j+1) = Temp_s(i,j+1);

        % Temperature profile in Polymer
        for i = nodes-1 : -1 : 2

            Temp_p(i,j+1) = Temp_p(i,j) + ((k_p/((-0.58 * Temp_p(i,j) + 903)*Cp_p))*dt/(r_p(i)*dr1^2))*(((r_p(i)+r_p(i-1))/2)*Temp_p(i-1,j)-2*r_p(i)*Temp_p(i,j)+((r_p(i+1)+r_p(i))/2)*Temp_p(i+1,j));

        end

        % Boundary Condition 5: @r=0, kdT/dr = 0
        i=1;
        Temp_p(i,j+1) = Temp_p(i,j) + (4*(k_p/((-0.58 * Temp_p(i,j) + 903)*Cp_p))*dt/dr1^2)*(Temp_p(i+1,j)-Temp_p(i,j));

```

```

    for i = 1 : nodes

        Temp_s(i,j) = Temp_s(i,j+1);
        Temp_p(i,j) = Temp_p(i,j+1);

    end

end

% Creating Temperature Profiles for Polymer and Solid
for i = 1 : nodes

    Temp_fp(i,k) = Temp_p(i,j);
    Temp_fs(i,k) = Temp_s(i,j);

end
end

%% Temperature Profile
figure
ai = Temp_fp(1:nodes,1);
bi = Temp_fp(1:nodes,20);
ci = Temp_fp(1:nodes,40);
di = Temp_fp(1:nodes,60);
ei = Temp_fp(1:nodes,80);
fi = Temp_fp(1:nodes,100);
plot(ai,r_p,'b',bi,r_p,'r',ci,r_p,'g',di,r_p,'c',ei,r_p,'m',fi,r_p,'y', ai(end:-1:1),↵
r_p'-0.00635, 'b', bi(end:-1:1),r_p'-0.00635,'r', ci(end:-1:1),r_p'-0.00635,'g', di(end:-1:1),↵
r_p'-0.00635,'c', ei(end:-1:1),r_p'-0.00635,'m', fi(end:-1:1),r_p'-0.00635,'y')
axis([50 275 -0.00635 0.00635 ])
xlabel('Temperature (^o)C')
ylabel('Radial Position (m)')
legend('t = 1 sec','t = 10 sec' , 't = 20 sec','t = 30 sec','t = 40 sec','t = 50 sec')

%% Viscosity Model Parameters
n = 0.2968;
tao_star = 1.738E4; %Pa
B = 1.6783E-6; %Pa-s
Tb = 1.1376E4 - 273.15; %degC

%initialize eta_o
eta_o = zeros(nodes,length(t));
%calculate eta_o
for k = 1 : length(t)
    for i = 1 : nodes
        eta_o(i,k) = B.*exp(Tb./Temp_fp(i,k));
    end
end

%% Velocity Profile Calculation

% Loading data from Mathematica solution:
% Note: the Mathematica solution solves for the DIMENSIONLESS velocity

% define search directory
search_directory = 'Mathematica_solutions';

```



```
% find files in directory
files = Find_Files(search_directory);
files = Extract_Files('CE_Model_PS_data.txt', files);

filename = strcat(search_directory, filesep, files);
filename = char(filename);
fid = fopen(filename);
filedata = textscan(fid, '%f%f', 'Delimiter', ' ', 'Headerlines', 0);
fclose(fid);

dim_r_p = cell2mat(filedata(:,1)); %normalized r_p
dim_vel = cell2mat(filedata(:,2)); %normalized velocity

%plot dimensionless velocity profile
figure
plot(dim_vel, dim_r_p*0.00635);
hold on
plot(dim_vel, -dim_r_p*0.00635)
ylabel('Radius')
xlabel('Dimensionless Velocity')
title('Dimensionless Velocity Profile for CE Model for PS')
axis([0 1 -0.00635 0.00635])

% solving for normalized velocity... eventually
% but first, non-normalized velocity,
% initialize velocity matrix
vel = zeros(nodes,length(t));
% velocity profile calculation
for k = 1 : length(t)
    for i = nodes-1 : -1 : 1

        vel(i,k) = dim_vel(i)*delta_p*R01^2/(L*eta_o(i,k));

    end
end

% Plot Velocity Profile
figure
hold on
plot(vel(1:157,1),dim_r_p)
plot(vel(1:157,10),dim_r_p)
plot(vel(1:157,20),dim_r_p)
plot(vel(1:157,30),dim_r_p)
plot(vel(1:157,40),dim_r_p)
plot(vel(1:157,50),dim_r_p)
plot(vel(1:157,60),dim_r_p)
plot(vel(1:157,70),dim_r_p)
plot(vel(1:157,80),dim_r_p)
plot(vel(1:157,90),dim_r_p)
plot(vel(1:157,100),dim_r_p)
ylabel(' Normalized Radius')
xlabel('Velocity')
title('Velocity Profile for CE Model for PS')
```

```

% Now that we can calculate vel_max we can find normalized velocity!
%normalizing the velocity profile
n_vel = zeros(nodes,length(t));

for k = 1 : length(t)

    for i = 1 : nodes

        n_vel(i,k) = vel(i,k) / vel(1,k);

    end

end

%% Plot Normalized Velocity Profile
figure
hold on
plot(n_vel(1:157,1),dim_r_p)
%plot(n_vel(1:157,10),dim_r_p)
plot(n_vel(1:157,20),dim_r_p)
%plot(n_vel(1:157,30),dim_r_p)
plot(n_vel(1:157,40),dim_r_p)
%plot(n_vel(1:157,50),dim_r_p)
plot(n_vel(1:157,60),dim_r_p)
%plot(n_vel(1:157,70),dim_r_p)
plot(n_vel(1:157,80),dim_r_p)
%plot(n_vel(1:157,90),dim_r_p)
plot(n_vel(1:157,100),dim_r_p)
plot(n_vel(1:157,1),-dim_r_p)
%plot(n_vel(1:157,10),dim_r_p)
plot(n_vel(1:157,20),-dim_r_p)
%plot(n_vel(1:157,30),dim_r_p)
plot(n_vel(1:157,40),-dim_r_p)
%plot(n_vel(1:157,50),dim_r_p)
plot(n_vel(1:157,60),-dim_r_p)
%plot(n_vel(1:157,70),dim_r_p)
plot(n_vel(1:157,80),-dim_r_p)
%plot(n_vel(1:157,90),dim_r_p)
plot(n_vel(1:157,100),-dim_r_p)
ylabel('Normalized Radius')
xlabel('Normalized Velocity')
title('Normalized Velocity Profile for CE Model for PS')
box on
legend('t=1 sec','t=20 sec','t=40 sec','t=60 sec','t=80 sec','t=100 sec')

%% Fractional Coverage Calculation

%initialize vectors
m = zeros(length(t),1);
Rx = zeros(length(t),1);
eta_avg = zeros(length(t),1);

% Rx calculation
for k = 1 : length(t)

    Rx(k) = (eta_o(1,k)*R01^4/eta_o(100,k))^(1/4);

```

```
%Fractional coverage when integrating through ru*
m(k) = 1 - (1 - m_n) * (Rx(k) / R01)^2;

end

%% Experimental Data

Fo_exp = [0.009290161 0.009361771 0.009319921 0.00923157 0.00912741 0.00878424 0.008636369
0.008528489 0.007097774 0.006588133 0.005670779 0.00480439 0.004039928 0 0 0 0.010646289
0.01123963 0.011360531 0.011388431 0.011343791 0.01123963 0.01102573 0.01085275 0.010718829
0.010227788 0.010034348 0.009548658 0.009240126 0.008843946 0.005331866 0.001957565 0.001186136
0.000718709 0.000435483 0.009763718 0.010065503 0.010091078 0.010046903 0.009961343 0.009831143
0.009605617 0.009436822 0.009310342 0.00869282 0.008356928 0.008006014 0.007669818 0.001254455
0.000748789 0.000446954 0.000266789 0.013721433 0.014074369 0.014123194 0.014102269 0.014035309
0.013923709 0.013566588 0.013449408 0.013030907 0.012869087 0.012549687 0.01221483 0.011892585
0.000900184 0.000436854 0.000212003 0.000102884 1.90411E-05 0.007187798 0.007228718 0.007172918
0.007070618 0.006955298 0.006810218 0.006430777 0.006317317 0.005789076 0.00551253 0.005229214
0.004961374 0.000574741 0 0 0.009706057 0.010012958 0.010078058 0.010096658 0.010078058
0.010029698 0.009843698 0.009778598 0.009538657 0.0092543 0.00905196 0.008857896 0.001638369
0.000986789 0.000594343 0.000357973 0.010386076 0.01071855 0.010758076 0.01072785 0.01065345
0.01053441 0.010322835 0.010162409 0.009446308 0.009120212 0.008779032 0.008451206 0.000800971
0.00035954 0.00016139 7.24447E-05 0.010082522 0.010394538 0.010424763 0.010385238 0.010303398
0.010176917 0.009790037 0.009665416 0.009055335 0.008722543 0.008375039 0.008041633 0.002343016
0.009265981 0.009445006 0.009414781 0.009314806 0.009184605 0.009009765 0.008526164 0.008377364
0.007670562 0.007294246 0.006905696 0.00653596 0.000510563 0.000135167 6.95478E-05 0.009419617
0.009409387 0.009330337 0.009204786 0.009070866 0.008907186 0.008490545 0.008367785 0.007802344
0.007509058 0.007209932 0.006928142 0.001532225 0.000979925 0.000626705 0.000400805 0.000256333
0.009973712 0.010556823 0.010673073 0.010696323 0.010647963 0.010540083 0.010321533 0.010145762
0.010009982 0.009513361 0.009318061 0.008928142 0.008517329 0.008118358 0.000379347 0.000187439
9.26151E-05 1.78759E-05];
m_exp = [0.664455133 0.660600195 0.659903584 0.653815611 0.643863879 0.643420578 0.6420786
0.641032095 0.63 0.6320329 0.630065371 0.6241699 0.61863233 0.527927299 0.511768978
0.498364331 0.689057396 0.692621509 0.692304866 0.689900398 0.68720811 0.68546886 0.685555
0.684666754 0.687869965 0.680615113 0.688518593 0.669690715 0.667162285 0.66521013 0.643249142
0.562980759 0.533264801 0.512741854 0.505587609 0.672333745 0.680039601 0.667376696 0.668113545
0.669887703 0.665519991 0.66042195 0.663392543 0.65819495 0.651606294 0.642900574 0.641887469
0.635561537 0.536369509 0.520796145 0.51029797 0.492728249 0.71303761 0.6994871 0.701769188
0.69999603 0.701257595 0.698408593 0.699423999 0.698485672 0.699962583 0.699467845 0.686372017
0.680490058 0.670616312 0.54996579 0.516916765 0.498031168 0.493316219 0.486415845 0.646852772
0.641527395 0.6402977 0.642550577 0.641688346 0.640829917 0.634794885 0.629447227 0.622278177
0.619749266 0.613333547 0.602327828 0.541584269 0.527157593 0.518919719 0.675408624 0.668449729
0.665198697 0.666624644 0.661579087 0.658701847 0.659627084 0.658785575 0.65831585 0.648579795
0.642704076 0.639065328 0.568488481 0.537996416 0.522383791 0.512449143 0.698873255 0.692935765
0.690407814 0.691673269 0.688984876 0.692774128 0.694173317 0.688442735 0.678472635 0.669053441
0.661023285 0.658290052 0.586277093 0.526014283 0.515202825 0.496022157 0.679814469 0.678385666
0.678443498 0.675245682 0.675144517 0.678017961 0.670001147 0.671366624 0.669771592 0.659192099
0.656571262 0.650316108 0.593335238 0.547726996 0.673297245 0.672809622 0.669223792 0.668933217
0.667955017 0.66403757 0.662242917 0.659374749 0.657487586 0.654273822 0.636189927 0.625708532
0.500466487 0.473828363 0.457995038 0.693084074 0.690318652 0.687879622 0.68319826 0.684136001
0.679103836 0.679561568 0.67289133 0.6701 0.66521 0.65222 0.6294 0.575210798 0.539712115
0.529919918 0.527618377 0.515450591 0.686706115 0.679323206 0.684293066 0.689236851 0.684060921
0.686483024 0.681287794 0.677176137 0.672125212 0.668292035 0.667041945 0.665278969 0.661918419
0.65875516 0.562243403 0.544177804 0.532056223 0.511946383];

%% Fourier Number
```

```
Fo = a_p*t/R01^2;

%% Plot Fo vs m
figure
semilogx(Fo, m, Fo_exp, m_exp, 'rs')
xlabel('Fo')
ylabel('m')
title('Polystyrene Model')
axis([1E-5 1E-1 0.45 0.9])
legend('Simulated Data', 'Experimental Data')
%*****
%
%                               CODE END
%
%*****
```

12 Appendix E: MATLAB code for PS: Ellis Model

11/1/17 8:36 AM C:\Users\estanisauskis...\Ellis Model PS TOT.m 1 of 11

```
*****
%
% PROJECT:      GAS ASSISTED INJECTION MOLDING
% PROGRAM NAME: Ellis_Model
% PURPOSE:      m vs Fo for PS
% DATE:         10/27/2017
%
%*****
clear variables
close all
clc
%% Import Mathematica Solution
% Loading data from Mathematica solution:
% Note: the Mathematica solution solves for the DIMENSIONLESS velocity

% define search directory
search_directory = 'Mathematica_solutions';

% find files in directory
filess = Find_Files(search_directory);
for indexx=4:length(filess)
    files = Extract_Files(filess(indexx), filess);

    filename = strcat(search_directory, filesep, files);
    filename = char(filename);
    fid = fopen(filename);
    filedata = textscan(fid, '%f%f', 'Delimiter', ' ', 'Headerlines', 0);
    fclose(fid);
    % factor = 140;
    if indexx == 4
        dim_r_p = cell2mat(filedata(:,1)); %dimensionless r_p
        dim_vel = -cell2mat(filedata(:,2)); %dimensionless velocity
    elseif indexx == 7
        dim_r_p = cell2mat(filedata(:,1)); %dimensionless r_p
        dim_vel = -cell2mat(filedata(:,2)); %dimensionless velocity
    elseif indexx == 8
        dim_r_p = cell2mat(filedata(:,1)); %dimensionless r_p
        dim_vel = -cell2mat(filedata(:,2)); %dimensionless velocity
    elseif indexx == 9
        dim_r_p = cell2mat(filedata(:,1)); %dimensionless r_p
        dim_vel = -cell2mat(filedata(:,2)); %dimensionless velocity
        dim_r_p = dim_r_p(2:2:end);
        dim_vel = dim_vel(2:2:end);
    elseif indexx == 5
        %shorten vectors TURN ON FOR DATA FILES 4 THROUGH 11
        dim_r_p = cell2mat(filedata(:,1)); %dimensionless r_p
        dim_vel = -cell2mat(filedata(:,2)); %dimensionless velocity
        dim_r_p = dim_r_p(2:2:end);
        dim_vel = dim_vel(2:2:end);
    elseif indexx == 6
        %shorten vectors TURN ON FOR DATA FILES 4 THROUGH 11
        dim_r_p = cell2mat(filedata(:,1)); %dimensionless r_p
        dim_vel = -cell2mat(filedata(:,2)); %dimensionless velocity
        dim_r_p = dim_r_p(2:2:end);
        dim_vel = dim_vel(2:2:end);
    elseif indexx == 10
```

```

%shorten vectors TURN ON FOR DATA FILES 4 THROUGH 11
dim_r_p = cell2mat(filedata(:,1)); %dimensionless r_p
dim_vel = -cell2mat(filedata(:,2)); %dimensionless velocity
dim_r_p = dim_r_p(2:2:end);
dim_vel = dim_vel(2:2:end);
elseif indexx == 11
%shorten vectors TURN ON FOR DATA FILES 4 THROUGH 11
dim_r_p = cell2mat(filedata(:,1)); %dimensionless r_p
dim_vel = -cell2mat(filedata(:,2)); %dimensionless velocity
dim_r_p = dim_r_p(2:2:end);
dim_vel = dim_vel(2:2:end);
elseif indexx == 12
dim_r_p = cell2mat(filedata(:,1)); %dimensionless r_p
dim_vel = -cell2mat(filedata(:,2)); %dimensionless velocity
dim_r_p = dim_r_p(2:2:end);
dim_vel = dim_vel(2:2:end);
elseif indexx ==13
%shorten vectors TURN ON FOR DATA FILES 4 THROUGH 11
dim_r_p = cell2mat(filedata(:,1)); %dimensionless r_p
dim_vel = -cell2mat(filedata(:,2)); %dimensionless velocity
dim_r_p = dim_r_p(2:2:end);
dim_vel = dim_vel(2:2:end);
elseif indexx ==14
%shorten vectors TURN ON FOR DATA FILES 4 THROUGH 11
dim_r_p = cell2mat(filedata(:,1)); %dimensionless r_p
dim_vel = -cell2mat(filedata(:,2)); %dimensionless velocity
dim_r_p = dim_r_p(2:2:end);
dim_vel = dim_vel(2:2:end);
end
factor = round(0.9*length(dim_vel));
dim_vel_plot(1:length(dim_vel),indexx) = dim_vel(1:length(dim_vel),1);
dim_r_p_plot(1:length(dim_r_p),indexx) = dim_r_p(1:length(dim_r_p),1);
%% Initial Conditions

% Number of nodes
nodes = length(dim_vel); %this number varies based on the Mathematica solution

% time measurements
t = linspace(1E-10,50,1000); %sec

% time increments
dt = 0.0001;

% Temperature of mold
Tb = (180-32)/1.8; %degC

% Temperature of melt
Ti = (623.6-32)/1.8; %degC

% Temperature of cooling water
Tc = 25; %degC

%Fractional coverage asymptotic value
n_p = 0.2968;
if n_p >= 0.3
    m_n = 0.1516*log(n_p)+0.7259;

```

```
else
    m_n = -0.1428*n_p^2+0.2626*n_p+0.4782;
end

%% Ellis model parameters

% a: (measure of degree of shear thinning behavior)
a = 3.26;

% Tao_half: (value of shear stress at which the apparent
% viscosity has dropped to half its zero shear value)
tao_half = 16618.6;

% Zero Shear Viscosity
% function of temperature
% defined later

%% Physical Properties of the tube

% External Diameter
OD = 2 * (150E-3+0.00635); %m

% Internal Diameter
ID = 2 * 0.00635; %m

% External Radius
R02 = OD * 0.5; %m

% Internal Radius
R01 = ID * 0.5; %m

% Length of tube
L = 0.585; %m

% Gravitational Acceleration
g = 9.81; %m/s^2

% Diameter
D = OD; %m

% Thermal Conductivity
k_s = 16; %W(m*K)

%Density
den_s = 7850; %kg/m^3

%Specific Heat
Cp_s = 514.5; %J/(kg*K)

% Thermal Diffusivity
a_s = k_s / (den_s * Cp_s); %m^2/s

% Step change in tube
dr2 = (R02 - R01) / (nodes - 1);

%% Physical Properties of the polymer
```

```
% Specific Heat
Cp_p = 2100; %J/(kg*K)

% Thermal Conductivity
k_p = 0.15; %W/(m*K)

% Density
den_p = 940; %kg/m^3

% Thermal Diffusivity
a_p = k_p / (den_p * Cp_p); %m^2/s

% Flow Activation Energy
%delta_H_R = 8575; %K
delta_H_R = 9.0679E3; %K

% Zero shear viscosity
zero_visc = 1*10^16*exp(-0.056*(Ti+273.13));

% Frequency factor
A = zero_visc / exp(delta_H_R / (25+273.13)); %kg/(m*s)

% Pressure gradient
delta_p = 2.42 * 10^6; %Pa

% Step change in polymer
dr1 = R01 / (nodes - 1);

n_dr1 = 1.0 / (nodes - 1);

%% Physical Properties of the cooling fluid

% Specific Heat
Cp_f = 10^-6 * Tb^4 - 0.0004 * Tb^3 + 0.0488 * Tb^2 - 2.2527 * Tb + 4212; %J/(kg*K)

% Viscosity
u_f = (4 * 10^-8 * Tb^4 - 10^-5 * Tb^3 + 0.001 * Tb^2 - 0.0557 * Tb + 1.7832) * 10^-3; %kg/(m*s)

% Density
den_f = -0.0036 * Tb^2 - 0.0656 * Tb + 1000.4; %kg/m^3

% Thermal Conductivity
k_f = -9 * 10^-6 * Tb^2 + 0.0021 * Tb + 0.5607; %W/(m*K)

% Coefficient of Volume Expansion
b = (0.0036 * 2 * Tb + 0.0656) / den_f; %1/K

% Thermal Diffusivity
a_f = k_f / (den_f * Cp_f); %m^2/s

%% Heat Transfer outside the tube wall

% Initializing temperature profiles
Temp_p = zeros(nodes,2);
```



```
Temp_s = zeros(nodes,2);
Temp_fp = zeros(nodes,length(t));
Temp_fs = zeros(nodes,length(t));

% Initializing radial position profiles
for i = nodes : -1 : 1
    r_s(i) = R01 + dr2 * (i-1);
end
for i = nodes : -1 : 1
    r_p(i) = dr1 * (i-1);
end

% Boundary Condition 1: @ t=0, T_f(r,0) = Ti
Temp_p = Temp_p + Ti;

% Boundary Condition 2: @ t=0, T_s(r,0) = Tc
Temp_s = Temp_s + Tc;

% Calculation at R01
i=nodes;
j=1;

% Grashof Number
Gr = D^3 * den_f^2 * g * b * (Temp_s(i,j)-Tb) / u_f^2;

% Prandtl Number
Pr = Cp_f * u_f / k_f;

% Raleigh Number
Ra = Gr * Pr;

% Reynolds Number
Re = 1;

% Nusselt Number
Nu = 0.62*Re^(1/2)*Pr^(1/3)/[1+(0.4/Pr)^(2/3)]^(1/4);

% Heat Transfer Coefficient
h = k_f * Nu / D;

%Initialization of counter variables
count = 0;

for k = 1 : length(t)

    disp(length(t)-k+1)

    while count <= t(k)

        %counter variables incrementation
        count = count + dt;

        % Boundary Condition 3: @ r=R02, -ks*dT/dr = h(Ts(R02,t)-Tb)
        i=nodes;
        j=1;
```

```

    Temp_s(i,j+1) = Temp_s(i,j) + (2*a_s*dt/dr2^2)*(Temp_s(i-1,j)-Temp_s(i,j)-((R02+
(dr2/2))/R02)*(h/k_s)*dr2*(Temp_s(i,j)-Tb));

    % Temperature profile in Solid
    for i = nodes-1 : -1 : 2

        Temp_s(i,j+1) = Temp_s(i,j) + (a_s*dt/(r_s(i)*dr2^2))*(((r_s(i)+r_s(i-1))/2)
*Temp_s(i-1,j)-2*r_s(i)*Temp_s(i,j)+((r_s(i+1)+r_s(i))/2)*Temp_s(i+1,j));

    end

    % Boundary Condition 4: @ r=R01, solid-liquid interface energy balance
    i = 1;
    Temp_s(i,j+1) = Temp_s(i,j) + (2*k_s*dt*(R01+dr2/2)*(Temp_s(i+1,j)-Temp_s(i,j))/(dr2*
((R01+dr2/4)*den_s*Cp_s*dr2+(R01-dr1/4)*(-0.58 * Temp_p(nodes,j) + 903)*Cp_p*dr1)))+(2*k_p*dt*
(R01-dr1/2)*(Temp_p(nodes-i,j)-Temp_s(i,j))/(dr1*((R01+dr2/4)*den_s*Cp_s*dr2+(R01-dr1/4)*(-0.58 *
Temp_p(nodes,j) + 903)*Cp_p*dr1)));

    Temp_p(nodes,j+1) = Temp_s(i,j+1);

    % Temperature profile in Polymer
    for i = nodes-1 : -1 : 2

        Temp_p(i,j+1) = Temp_p(i,j) + ((k_p/((-0.58 * Temp_p(i,j) + 903)*Cp_p))*dt/(r_p
(i)*dr1^2))*(((r_p(i)+r_p(i-1))/2)*Temp_p(i-1,j)-2*r_p(i)*Temp_p(i,j)+((r_p(i+1)+r_p(i))/2)
*Temp_p(i+1,j));

    end

    % Boundary Condition 5: @r=0, kdT/dr = 0
    i=1;
    Temp_p(i,j+1) = Temp_p(i,j) + (4*(k_p/((-0.58 * Temp_p(i,j) + 903)*Cp_p))*dt/dr1^2)*
(Temp_p(i+1,j)-Temp_p(i,j));

    for i = 1 : nodes

        Temp_s(i,j) = Temp_s(i,j+1);
        Temp_p(i,j) = Temp_p(i,j+1);

    end

end

% Creating Temperature Profiles for Polymer and Solid
for i = 1 : nodes

    Temp_fp(i,k) = Temp_p(i,j);
    Temp_fs(i,k) = Temp_s(i,j);

end

end

%% Temperature Profile Calculation
% figure
% plot(r_p,Temp_fp(1:nodes,4))
% hold on

```

```
% plot(r_p,Temp_fp(1:nodes,5))
% plot(r_p,Temp_fp(1:nodes,7))

%% Velocity Profile Calculation

%initialize eta_o
eta_o = zeros(nodes,length(t));
%calculate eta_o
for k = 1 : length(t)
    for i = 1 : nodes
        eta_o(i,k) = 1*10^16*exp(-0.056*(Temp_fp(i,k)+273.13));
    end
end

% THE INPUT OF THE MATHEMATICA SOLUTION IS IMPORTANT FOR THE FOLLOWING
% PORTION OF CODE

%plot dimensionless velocity profile
figure
plot(dim_vel, dim_r_p*0.00635);
hold on
plot(dim_vel, -dim_r_p*0.00635)
ylabel('Radius')
xlabel('Dimensionless Velocity')
title('Dimensionless Velocity Profile for Ellis Model for PS')
axis([0 1 -0.00635 0.00635])

% solving for normalized velocity... eventually
% but first, non-normalized velocity,
% initialize velocity matrix
vel = zeros(nodes,length(t));
% velocity profile calculation
for k = 1 : length(t)
    for i = nodes-1 : -1 : 1

        vel(i,k) = dim_vel(i)*delta_p*R01^2/(L*eta_o(i,k));

    end
end

% Plot Velocity Profile
figure
hold on
plot(vel(1:nodes,1),dim_r_p)
plot(vel(1:nodes,10),dim_r_p)
plot(vel(1:nodes,20),dim_r_p)
plot(vel(1:nodes,30),dim_r_p)
plot(vel(1:nodes,40),dim_r_p)
plot(vel(1:nodes,50),dim_r_p)
plot(vel(1:nodes,60),dim_r_p)
plot(vel(1:nodes,70),dim_r_p)
plot(vel(1:nodes,80),dim_r_p)
plot(vel(1:nodes,90),dim_r_p)
plot(vel(1:nodes,100),dim_r_p)
plot(vel(1:nodes,1),-dim_r_p)
```

```
plot(vel(1:nodes,10),-dim_r_p)
plot(vel(1:nodes,20),-dim_r_p)
plot(vel(1:nodes,30),-dim_r_p)
plot(vel(1:nodes,40),-dim_r_p)
plot(vel(1:nodes,50),-dim_r_p)
plot(vel(1:nodes,60),-dim_r_p)
plot(vel(1:nodes,70),-dim_r_p)
plot(vel(1:nodes,80),-dim_r_p)
plot(vel(1:nodes,90),-dim_r_p)
plot(vel(1:nodes,100),-dim_r_p)
ylabel(' Normalized Radius')
xlabel('Velocity')
title('Velocity Profile for CE Model for PS')

% Now that we can calculate vel_max we can find normalized velocity!
% normalizing the velocity profile
n_vel = zeros(nodes,length(t));

for k = 1 : length(t)

    for i = 1 : nodes

        n_vel(i,k) = vel(i,k) / vel(1,k);

    end

end

% Plot Normalized Velocity Profile
figure
hold on
plot(n_vel(1:nodes,10),dim_r_p)
%plot(n_vel(1:157,10),dim_r_p)
plot(n_vel(1:nodes,200),dim_r_p)
%plot(n_vel(1:157,30),dim_r_p)
plot(n_vel(1:nodes,400),dim_r_p)
%plot(n_vel(1:157,50),dim_r_p)
plot(n_vel(1:nodes,600),dim_r_p)
%plot(n_vel(1:157,70),dim_r_p)
plot(n_vel(1:nodes,800),dim_r_p)
%plot(n_vel(1:157,90),dim_r_p)
plot(n_vel(1:nodes,1000),dim_r_p)
plot(n_vel(1:nodes,10),-dim_r_p)
%plot(n_vel(1:157,10),-dim_r_p)
plot(n_vel(1:nodes,200),-dim_r_p)
%plot(n_vel(1:157,30),-dim_r_p)
plot(n_vel(1:nodes,400),-dim_r_p)
%plot(n_vel(1:157,50),-dim_r_p)
plot(n_vel(1:nodes,600),-dim_r_p)
%plot(n_vel(1:157,70),-dim_r_p)
plot(n_vel(1:nodes,800),-dim_r_p)
%plot(n_vel(1:157,90),-dim_r_p)
plot(n_vel(1:nodes,100),-dim_r_p)
ylabel('Normalized Radius')
xlabel('Normalized Velocity')
title('Normalized Velocity Profile for Ellis Model for PS')
```

```

box on
legend('t=1 sec','t=20 sec','t=40 sec','t=60 sec','t=80 sec','t=100 sec')

end
%% Plot All Dimensionless Velocity Profiles
figure
plot(dim_vel_plot(1:173,4),dim_r_p_plot(1:173,4))
hold on
plot(dim_vel_plot(1:173,4),-dim_r_p_plot(1:173,4))
% title('\tau_{wall}=1.47E6 Pa')
ylabel('dimensionless radius')
xlabel('dimensionless velocity')
for i = 5:length(files)
    plot(dim_vel_plot(1:173,i),dim_r_p_plot(1:173,i))
end
for i = 5:length(files)
    plot(dim_vel_plot(1:173,i),-dim_r_p_plot(1:173,i))
end
title('All Dimensionless Velocity Profiles')

%% Fractional Coverage Calculation

%initialize vectors
m = zeros(length(t),1);
Rx = zeros(length(t),1);

%define shear stress average and wall
tau_avg = linspace(1.47E6,3.42,length(t));
tau_wall = tau_avg(1); %isothermal

% Rx calculation
for k = 1 : length(t)

    Rx(k) = R01*sqrt(tau_avg(k)*eta_o(1,k)/(tau_wall*eta_o(105,k)));

    %Fractional coverage when integrating through ru*
    m(k) = 1 - (1 - m_n) * (Rx(k) / R01)^2;

end

%% Experimental Data

Fo_exp = [0.009290161 0.009361771 0.009319921 0.00923157 0.00912741 0.00878424 ✓
0.008636369 0.008528489 0.007097774 0.006588133 0.005670779 0.00480439 0.004039928 0 0 0 ✓
0.010646289 0.01123963 0.011360531 0.011388431 0.011343791 0.01123963 0.01102573 0.01085275 ✓
0.010718829 0.010227788 0.010034348 0.009548658 0.009240126 0.008843946 0.005331866 0.001957565 ✓
0.001186136 0.000718709 0.000435483 0.009763718 0.010065503 0.010091078 0.010046903 0.009961343 ✓
0.009831143 0.009605617 0.009436822 0.009310342 0.00869282 0.008356928 0.008006014 0.007669818 ✓
0.001254455 0.000748789 0.000446954 0.000266789 0.013721433 0.014074369 0.014123194 0.014102269 ✓
0.014035309 0.013923709 0.013566588 0.013449408 0.013030907 0.012869087 0.012549687 0.01221483 ✓
0.011892585 0.000900184 0.000436854 0.000212003 0.000102884 1.90411E-05 0.007187798 0.007228718 ✓
0.007172918 0.007070618 0.006955298 0.006810218 0.006430777 0.006317317 0.005789076 0.00551253 ✓
0.005229214 0.004961374 0.000574741 0 0 0.009706057 0.010012958 0.010078058 0.010096658 ✓
0.010078058 0.010029698 0.009843698 0.009778598 0.009538657 0.0092543 0.00905196 0.008857896 ✓
0.001638369 0.000986789 0.000594343 0.000357973 0.010386076 0.01071855 0.010758076 0.01072785 ✓

```

```

0.01065345 0.01053441 0.010322835 0.010162409 0.009446308 0.009120212 0.008779032 0.008451206 ✓
0.000800971 0.00035954 0.00016139 7.24447E-05 0.010082522 0.010394538 0.010424763 0.010385238 ✓
0.010303398 0.010176917 0.009790037 0.009665416 0.009055335 0.008722543 0.008375039 0.008041633 ✓
0.002343016 0 0.009265981 0.009445006 0.009414781 0.009314806 0.009184605 0.009009765 ✓
0.008526164 0.008377364 0.007670562 0.007294246 0.006905696 0.00653596 0.000510563 0.000135167 ✓
6.95478E-05 0.009419617 0.009409387 0.009330337 0.009204786 0.009070866 0.008907186 0.008490545 ✓
0.008367785 0.007802344 0.007509058 0.007209932 0.006928142 0.001532225 0.000979925 0.000626705 ✓
0.000400805 0.000256333 0.009973712 0.010556823 0.010673073 0.010696323 0.010647963 0.010540083 ✓
0.010321533 0.010145762 0.010009982 0.009513361 0.009318061 0.008928142 0.008517329 0.008118358 ✓
0.000379347 0.000187439 9.26151E-05 1.78759E-05];
m_exp = [0.664455133 0.660600195 0.659903584 0.653815611 0.643863879 0.643420578 0.6420786 ✓
0.641032095 0.63 0.6320329 0.630065371 0.6241699 0.61863233 0.527927299 0.511768978 ✓
0.498364331 0.689057396 0.692621509 0.692304866 0.689900398 0.68720811 0.68546886 0.685555 ✓
0.684666754 0.687869965 0.680615113 0.688518593 0.669690715 0.667162285 0.66521013 0.643249142 ✓
0.562980759 0.533264801 0.512741854 0.505587609 0.672333745 0.680039601 0.667376696 0.668113545 ✓
0.669887703 0.665519991 0.66042195 0.663392543 0.65819495 0.651606294 0.642900574 0.641887469 ✓
0.635561537 0.536369509 0.520796145 0.51029797 0.492728249 0.71303761 0.6994871 0.701769188 ✓
0.69999603 0.701257595 0.698408593 0.699423999 0.698485672 0.699962583 0.699467845 0.686372017 ✓
0.680490058 0.670616312 0.54996579 0.516916765 0.498031168 0.493316219 0.486415845 0.646852772 ✓
0.641527395 0.6402977 0.642550577 0.641688346 0.640829917 0.634794885 0.629447227 0.622278177 ✓
0.619749266 0.613333547 0.602327828 0.541584269 0.527157593 0.518919719 0.675408624 0.668449729 ✓
0.665198697 0.666624644 0.661579087 0.658701847 0.659627084 0.658785575 0.65831585 0.648579795 ✓
0.642704076 0.639065328 0.568488481 0.537996416 0.522383791 0.512449143 0.698873255 0.692935765 ✓
0.690407814 0.691673269 0.688984876 0.692774128 0.694173317 0.688442735 0.678472635 0.669053441 ✓
0.661023285 0.658290052 0.586277093 0.526014283 0.515202825 0.496022157 0.679814469 0.678385666 ✓
0.678443498 0.675245682 0.675144517 0.678017961 0.670001147 0.671366624 0.669771592 0.659192099 ✓
0.656571262 0.650316108 0.593335238 0.547726996 0.673297245 0.672809622 0.669223792 0.668933217 ✓
0.667955017 0.66403757 0.662242917 0.659374749 0.657487586 0.654273822 0.636189927 0.625708532 ✓
0.500466487 0.473828363 0.457995038 0.693084074 0.690318652 0.687879622 0.68319826 0.684136001 ✓
0.679103836 0.679561568 0.67289133 0.6701 0.66521 0.65222 0.6294 0.575210798 0.539712115 ✓
0.529919918 0.527618377 0.515450591 0.686706115 0.679323206 0.684293066 0.689236851 0.684060921 ✓
0.686483024 0.681287794 0.677176137 0.672125212 0.668292035 0.667041945 0.665278969 0.661918419 ✓
0.65875516 0.562243403 0.544177804 0.532056223 0.511946383];

```

```
%% Fourier Number
```

```
Fo = a_p*t/R01^2;
```

```
%% Plot Fractional Coverage vs Fourier Number
figure
```

```
semilogx(Fo, m, Fo_exp, m_exp, 'rs')
```

```
xlabel('Fo')
```

```
ylabel('m')
```

```
title('Polystyrene Ellis Model')
```

```
legend('Simulated Data', 'Experimental Data')
```

```
axis([3E-5 3E-2 0.4 0.9])
```

```
fprintf('--End--\n')
```

```
%*****
%
```

```
%  
% CODE END  
%*****
```

13 Appendix F: MATLAB code for PS: Power Law Model

11/15/17 11:03 AM C:\Users\estanisauskis\Deskt...\Power Law PS.m 1 of 9

```
*****
%
% PROJECT:      GAS ASSISTED INJECTION MOLDING
% PROGRAM NAME: Power Law Model Simulation vs Experimental Data
% PURPOSE:      Power Law Simulation for PS
% DATE:         11/15/2017
%
*****
% clear variables
close all
clc
%% Load Excel Files
% piston speed data references
% % data_points = [2 37 18 9 45 26 50 33 17 53 36];

% delay time
% % data_points = [54 37 72 61 45 79 67 50 84 70 53];

% gas pressure
% % data_points = [87 37 103 94 45 110 99 50 115 102 53 118];

% shot size
% % data_points = [119 37 133 126 45 140 131 50 145 53 147];

% melt temperature
data_points = [148 37 165 155 45 173 160 50 163 53 181];

data_points = data_points-1;

Ti_dp = xlsread('PS_Fractional_Cov_Comp.xlsx','I2:I182'); %degC
m_exp_xls = xlsread('PS_Fractional_Cov_Comp.xlsx','J2:J182');

for data_point_no = 1:length(data_points)
    % find files
    convertedrestime = xlsread('PS_Fractional_Cov_Comp.xlsx','F2:F182');
    convertedrowlength = xlsread('PS_Fractional_Cov_Comp.xlsx','G2:G182');

    dp = data_points(data_point_no);

    converted_res_time=zeros(length(data_points),1);
    converted_rowlength=zeros(length(data_points),1);
    % factor=zeros(length(data_points),1);

    for i = 1:length(data_points)
        dp = data_points(i);
        converted_res_time(i) = convertedrestime(dp);
        converted_rowlength(i) = convertedrowlength(dp);
        % factor(i) = factorr(dp);
    end
    %% Initial Conditions

    % Number of nodes
    nodes = 21;

    % time measurements
    t = linspace(1E-10,50,1000); %sec
```



```
% time increments
dt = 0.0001;

% Temperature of mold
Tb = (180-32)/1.8; %degC

% Temperature of melt
Ti = (623.6-32)/1.8; %degC

% Temperature of cooling water
Tc = 25; %degC

%Fractional coverage asymptotic value
n_p = 0.2968;
if n_p >= 0.3
    m_n = 0.1516*log(n_p)+0.7259;
else
    m_n = -0.1428*n_p^2+0.2626*n_p+0.4782;
end

%% Physical Properties of the tube

% External Diameter
OD = 2 * (150E-3+0.00635); %m

% Internal Diameter
ID = 2 * 0.00635; %m

% External Radius
R02 = OD * 0.5; %m

% Internal Radius
R01 = ID * 0.5; %m

% Length of tube
L = 0.585; %m

% Gravitational Acceleration
g = 9.81; %m/s^2

% Diameter
D = OD; %m

% Thermal Conductivity
k_s = 16; %W(m*K)

%Density
den_s = 7850; %kg/m^3

%Specific Heat
Cp_s = 514.5; %J/(kg*K)

% Thermal Diffusivity
a_s = k_s / (den_s * Cp_s); %m^2/s
```

```
% Step change in tube
dr2 = (R02 - R01) / (nodes - 1);

%% Physical Properties of the polymer

% Specific Heat
Cp_p = 2100; %J/(kg*K)

% Thermal Conductivity
k_p = 0.15; %W/(m*K)

% Density
den_p = 940; %kg/m^3

% Thermal Diffusivity
a_p = k_p / (den_p * Cp_p); %m^2/s

% Flow Activation Energy
%delta_H_R = 8575; %K
delta_H_R = 9.0679E3; %K

% Zero shear viscosity
zero_visc = 1*10^16*exp(-0.056*(Ti+273.13));

% Frequency factor
A = zero_visc / exp(delta_H_R / (25+273.13)); %kg/(m*s)

% Pressure gradient
delta_p = 2.42 * 10^6; %Pa

% Step change in polymer
dr1 = R01 / (nodes - 1);

n_dr1 = 1.0 / (nodes - 1);

%% Physical Properties of the cooling fluid

% Specific Heat
Cp_f = 10^-6 * Tb^4 - 0.0004 * Tb^3 + 0.0488 * Tb^2 - 2.2527 * Tb + 4212; %J/(kg*K)

% Viscosity
u_f = (4 * 10^-8 * Tb^4 - 10^-5 * Tb^3 + 0.001 * Tb^2 - 0.0557 * Tb + 1.7832) * 10^-3; %kg/(m*s)

% Density
den_f = -0.0036 * Tb^2 - 0.0656 * Tb + 1000.4; %kg/m^3

% Thermal Conductivity
k_f = -9 * 10^-6 * Tb^2 + 0.0021 * Tb + 0.5607; %W/(m*K)

% Coefficient of Volume Expansion
b = (0.0036 * 2 * Tb + 0.0656) / den_f; %1/K

% Thermal Diffusivity
a_f = k_f / (den_f * Cp_f); %m^2/s
```

```
% Heat Transfer outside the tube wall

% Initializing temperature profiles
Temp_p = zeros(nodes,2);
Temp_s = zeros(nodes,2);
Temp_fp = zeros(nodes,length(t));
Temp_fs = zeros(nodes,length(t));

% Initializing radial position profiles
for i = nodes : -1 : 1
    r_s(i) = R01 + dr2 * (i-1);
end
for i = nodes : -1 : 1
    r_p(i) = dr1 * (i-1);
end

% Boundary Condition 1: @ t=0, T_f(r,0) = Ti
Temp_p = Temp_p + Ti;

% Boundary Condition 2: @ t=0, T_s(r,0) = Tc
Temp_s = Temp_s + Tc;

% Calculation at R01
i=nodes;
j=1;

% Grashof Number
Gr = D^3 * den_f^2 * g * b * (Temp_s(i,j)-Tb) / u_f^2;

% Prandtl Number
Pr = Cp_f * u_f / k_f;

% Raleigh Number
Ra = Gr * Pr;

% Reynolds Number
Re = 1;

% Nusselt Number
Nu = 0.62*Re^(1/2)*Pr^(1/3)/[1+(0.4/Pr)^(2/3)]^(1/4);

% Heat Transfer Coefficient
h = k_f * Nu / D;

%Initialization of counter variables
count = 0;

for k = 1 : length(t)

    disp(length(t)-k+1)

    while count <= t(k)

        %counter variables incrementation
```

```

count = count + dt;

% Boundary Condition 3: @ r=R02, -ks*dt/dr = h(Ts(R02,t)-Tb)
i=nodes;
j=1;
Temp_s(i,j+1) = Temp_s(i,j) + (2*a_s*dt/dr2^2)*(Temp_s(i-1,j)-Temp_s(i,j)-((R02+
(dr2/2))/R02)*(h/k_s)*dr2*(Temp_s(i,j)-Tb)));

% Temperature profile in Solid
for i = nodes-1 : -1 : 2

    Temp_s(i,j+1) = Temp_s(i,j) + (a_s*dt/(r_s(i)*dr2^2))*(((r_s(i)+r_s(i-1))/2)
*Temp_s(i-1,j)-2*r_s(i)*Temp_s(i,j)+((r_s(i+1)+r_s(i))/2)*Temp_s(i+1,j));

end

% Boundary Condition 4: @ r=R01, solid-liquid interface energy balance
i = 1;
Temp_s(i,j+1) = Temp_s(i,j) + (2*k_s*dt*(R01+dr2/2)*(Temp_s(i+1,j)-Temp_s(i,j))/(dr2*
(R01+dr2/4)*den_s*Cp_s*dr2+(R01-dr1/4)*(-0.58 * Temp_p(nodes,j) + 903)*Cp_p*dr1)))+(2*k_p*dt*
(R01-dr1/2)*(Temp_p(nodes-i,j)-Temp_s(i,j))/(dr1*((R01+dr2/4)*den_s*Cp_s*dr2+(R01-dr1/4)*(-0.58 *
Temp_p(nodes,j) + 903)*Cp_p*dr1)));

Temp_p(nodes,j+1) = Temp_s(i,j+1);

% Temperature profile in Polymer
for i = nodes-1 : -1 : 2

    Temp_p(i,j+1) = Temp_p(i,j) + ((k_p/((-0.58 * Temp_p(i,j) + 903)*Cp_p))*dt/(r_p
(i)*dr1^2))*(((r_p(i)+r_p(i-1))/2)*Temp_p(i-1,j)-2*r_p(i)*Temp_p(i,j)+((r_p(i+1)+r_p(i))/2)
*Temp_p(i+1,j)));

end

% Boundary Condition 5: @r=0, kdT/dr = 0
i=1;
Temp_p(i,j+1) = Temp_p(i,j) + (4*(k_p/((-0.58 * Temp_p(i,j) + 903)*Cp_p))*dt/dr1^2)*
(Temp_p(i+1,j)-Temp_p(i,j));

for i = 1 : nodes

    Temp_s(i,j) = Temp_s(i,j+1);
    Temp_p(i,j) = Temp_p(i,j+1);

end

end

% Creating Temperature Profiles for Polymer and Solid
for i = 1 : nodes

    Temp_fp(i,k) = Temp_p(i,j);
    Temp_fs(i,k) = Temp_s(i,j);

end
end

```

```
%% Velocity Profile Calculation

%initialize velocity matrix
vel = zeros(nodes,length(t));

%velocity profile calculation
for k = 1 : length(t)

    %equation 4.33
    vel(nodes,k) = 0;

    for i = nodes-1 : -1 : 1

        f(i) = (n_dr1/2)*delta_p/(2*L*A)*((r_p(i+1)/exp(delta_H_R/(Temp_fp(i+1,k)+273.13)))+(r_p(i)/exp(delta_H_R/(Temp_fp(i,k)+273.13))));
        vel(i,k) = vel(i+1,k) + f(i);

    end

end

%normalizing the velocity profile
n_vel = zeros(nodes,length(t));

for k = 1 : length(t)

    for i = 1 : nodes

        n_vel(i,k) = vel(i,k) / vel(1,k);

    end

end

%normalizing the radial position
n_radius = zeros(2*nodes,1);

for i = 1 : nodes
    n_radius(i) = r_p(i) / R01;
end

for i = 1 : nodes
    n_radius(i+nodes) = r_s(i) / R01;
end

%% Fractional Coverage Calculation

%initialize vectors
m = zeros(length(t),1);
Rx = zeros(length(t),1);

for k = 1 : length(t)

    %initializing summation variables
    sum = 0;
```

```

for i = 1 : nodes-1

    %integrate through ru*
    sum = sum + (n_dr1 / 2) * (n_radius(i)*n_vel(i,k) + n_radius(i+1)*n_vel(i+1,k));

end

%integrate through ru*
Rx(k) = R01 * sqrt(4 * sum);

%Fractional coverage when integrating through ru*
m(k) = 1 - (1 - m_n) * (Rx(k) / R01)^2-0.02;

end

%% Experimental Data

Fo_exp = [0.009290161 0.009361771 0.009319921 0.00923157 0.00912741 0.00878424 ✓
0.008636369 0.008528489 0.007097774 0.006588133 0.005670779 0.00480439 0.004039928 0 0 0 ✓
0.010646289 0.01123963 0.011360531 0.011388431 0.011343791 0.01123963 0.01102573 0.01085275 ✓
0.010718829 0.010227788 0.010034348 0.009548658 0.009240126 0.008843946 0.005331866 0.001957565 ✓
0.001186136 0.000718709 0.000435483 0.009763718 0.010065503 0.010091078 0.010046903 0.009961343 ✓
0.009831143 0.009605617 0.009436822 0.009310342 0.00869282 0.008356928 0.008006014 0.007669818 ✓
0.001254455 0.000748789 0.000446954 0.000266789 0.013721433 0.014074369 0.014123194 0.014102269 ✓
0.014035309 0.013923709 0.013566588 0.013449408 0.013030907 0.012869087 0.012549687 0.01221483 ✓
0.011892585 0.000900184 0.000436854 0.000212003 0.000102884 1.90411E-05 0.007187798 0.007228718 ✓
0.007172918 0.007070618 0.006955298 0.006810218 0.006430777 0.006317317 0.005789076 0.00551253 ✓
0.005229214 0.004961374 0.000574741 0 0 0.009706057 0.010012958 0.010078058 0.010096658 ✓
0.010078058 0.010029698 0.009843698 0.009778598 0.009538657 0.0092543 0.00905196 0.008857896 ✓
0.001638369 0.000986789 0.000594343 0.000357973 0.010386076 0.01071855 0.010758076 0.01072785 ✓
0.01065345 0.01053441 0.010322835 0.010162409 0.009446308 0.009120212 0.008779032 0.008451206 ✓
0.000800971 0.00035954 0.00016139 7.24447E-05 0.010082522 0.010394538 0.010424763 0.010385238 ✓
0.010303398 0.010176917 0.009790037 0.009665416 0.009055335 0.008722543 0.008375039 0.008041633 ✓
0.002343016 0 0.009265981 0.009445006 0.009414781 0.009314806 0.009184605 0.009009765 ✓
0.008526164 0.008377364 0.007670562 0.007294246 0.006905696 0.00653596 0.000510563 0.000135167 ✓
6.95478E-05 0.009419617 0.009409387 0.009330337 0.009204786 0.009070866 0.008907186 0.008490545 ✓
0.008367785 0.007802344 0.007509058 0.007209932 0.006928142 0.001532225 0.000979925 0.000626705 ✓
0.000400805 0.000256333 0.009973712 0.010556823 0.010673073 0.010696323 0.010647963 0.010540083 ✓
0.010321533 0.010145762 0.010009982 0.009513361 0.009318061 0.008928142 0.008517329 0.008118358 ✓
0.000379347 0.000187439 9.26151E-05 1.78759E-05];

m_exp = [0.664455133 0.660600195 0.659903584 0.653815611 0.643863879 0.643420578 0.6420786 ✓
0.641032095 0.63 0.6320329 0.630065371 0.6241699 0.61863233 0.527927299 0.511768978 ✓
0.498364331 0.689057396 0.692621509 0.692304866 0.689900398 0.68720811 0.68546886 0.685555 ✓
0.684666754 0.687869965 0.680615113 0.688518593 0.669690715 0.667162285 0.66521013 0.643249142 ✓
0.562980759 0.533264801 0.512741854 0.505587609 0.672333745 0.680039601 0.667376696 0.668113545 ✓
0.669887703 0.665519991 0.66042195 0.663392543 0.65819495 0.651606294 0.642900574 0.641887469 ✓
0.635561537 0.536369509 0.520796145 0.51029797 0.492728249 0.71303761 0.6994871 0.701769188 ✓
0.69999603 0.701257595 0.698408593 0.699423999 0.698485672 0.699962583 0.699467845 0.686372017 ✓
0.680490058 0.670616312 0.54996579 0.516916765 0.498031168 0.493316219 0.486415845 0.646852772 ✓
0.641527395 0.6402977 0.642550577 0.641688346 0.640829917 0.634794885 0.629447227 0.622278177 ✓
0.619749266 0.613333547 0.602327828 0.541584269 0.527157593 0.518919719 0.675408624 0.668449729 ✓
0.665198697 0.666624644 0.661579087 0.658701847 0.659627084 0.658785575 0.65831585 0.648579795 ✓
0.642704076 0.639065328 0.568488481 0.537996416 0.522383791 0.512449143 0.698873255 0.692935765 ✓
0.690407814 0.691673269 0.688984876 0.692774128 0.694173317 0.688442735 0.678472635 0.669053441 ✓

```

```

0.661023285 0.658290052 0.586277093 0.526014283 0.515202825 0.496022157 0.679814469 0.678385666 ✓
0.678443498 0.675245682 0.675144517 0.678017961 0.670001147 0.671366624 0.669771592 0.659192099 ✓
0.656571262 0.650316108 0.593335238 0.547726996 0.673297245 0.672809622 0.669223792 0.668933217 ✓
0.667955017 0.66403757 0.662242917 0.659374749 0.657487586 0.654273822 0.636189927 0.625708532 ✓
0.500466487 0.473828363 0.457995038 0.693084074 0.690318652 0.687879622 0.68319826 0.684136001 ✓
0.679103836 0.679561568 0.67289133 0.6701 0.66521 0.65222 0.6294 0.575210798 0.539712115 ✓
0.529919918 0.527618377 0.515450591 0.686706115 0.679323206 0.684293066 0.689236851 0.684060921 ✓
0.686483024 0.681287794 0.677176137 0.672125212 0.668292035 0.667041945 0.665278969 0.661918419 ✓
0.65875516 0.562243403 0.544177804 0.532056223 0.511946383];

```

```

%% Fourier Number

```

```

Fo = a_p*t/R01^2;

```

```

%% Determination of shear rate

```

```

if round(converted_res_time(data_point_no)) == 0
    shear_rate(data_point_no)=0;
else
    R_bubble = Rx(round(converted_res_time(data_point_no)));
    Q_exp = xlsread('PS_Fractional_Cov_Comp.xlsx','E2:E182');
    factor = Q_exp(data_points(data_point_no))/(3.14*R_bubble^2*n_vel(1,round(converted_res_time
(data_point_no))));
    shear_rate(data_point_no) = (factor*n_vel(1,round(converted_res_time(data_point_no)))-
factor*n_vel(2,round(converted_res_time(data_point_no))))/(t(2)-t(1));
end

```

```

%% Plot Viscosity vs shear rate

```

```

viscosity(data_point_no) = A*shear_rate(data_point_no)^(n_p-1);

```

```

sr_vector = logspace(-5, 10, 1E2);
visc = 0.0000000004*sr_vector.^(n_p-1);

```

```

figure
loglog(sr_vector,visc)
hold on
loglog(shear_rate,viscosity,'rs')

```

```

%% Plot m_sim vs m_exp

```

```

if round(converted_res_time(data_point_no))==0
    m_sim(data_point_no)=0;
    m_exp_dp(data_point_no)=0;
else
    figure

    m_exp_dp(data_point_no) = m_exp_xls(data_points(data_point_no));
    m_sim_dp(data_point_no) = m(round(converted_res_time(data_point_no)));
    plot(m_sim_dp,m_exp_dp,'rs')
    xlabel('m_{sim}')
    ylabel('m_{exp}')
    hold on
    line = linspace(0.5,1,100);
    plot(line,line)
end

```

```

end

```

```
%% PLOT RESULTS
% sim vs exp
figure
line = linspace(0.1,1,100);
plot(line,line)
% axis([0.4 1 0.4 1])
xlabel('m_{sim}')
ylabel('m_{exp}')
legend('Perfect Simumlation','High 90mm','Base 90mm','Low 90mm','High 285mm','Base 285mm','Low
285mm','High 445mm','Base 445mm','Low 445mm','Base 490mm','Low 490mm');
title('Varying Shot Size Setting')
axis([0.4 1 0.4 1])
box on
legend boxoff
legend('Location','NorthWest')
marker_point =
['s','>','x','s','>','x','s','>','x','s','>','x','s','>','x','s','>','x','s','>','x','s','>','x','s','>','x','s','>','x','s','>','x'];
hold on
for i = 1:length(m_sim_dp)
    plot(m_sim_dp(i),m_exp_dp(i),marker_point(i))
end
%% velocity profile
figure
plot(n_vel(1:21,1),n_radius(1:21))
hold on

plot(n_vel(1:21,200),n_radius(1:21))
plot(n_vel(1:21,400),n_radius(1:21))
plot(n_vel(1:21,600),n_radius(1:21))
plot(n_vel(1:21,800),n_radius(1:21))
plot(n_vel(1:21,1000),n_radius(1:21))
plot(n_vel(1:21,1),-n_radius(1:21))
plot(n_vel(1:21,200),-n_radius(1:21))
plot(n_vel(1:21,400),-n_radius(1:21))
plot(n_vel(1:21,600),-n_radius(1:21))
plot(n_vel(1:21,800),-n_radius(1:21))
plot(n_vel(1:21,1000),-n_radius(1:21))
xlabel('Normalized Velocity')
ylabel('Normalized Radius')
legend('t=0s','t=10s','t=20s','t=30s','t=40s','t=50s')
%*****
%
%
%
%
%*****
```


14 Appendix G: MATLAB code for PS Simulation Comparison

11/1/17 9:33 PM C:\Users\estanisauski...\Ellis Model Sim Exp.m 1 of 15

```
*****
%
% PROJECT:          GAS ASSISTED INJECTION MOLDING
% PROGRAM NAME:     Ellis_Model Simulation vs Experimental Data
% PURPOSE:          m vs Fo for PS
% DATE:             11/1/2017
%
*****
% clear variables
close all
clc
%% Import Mathematica Solution
% Loading data from Mathematica solution:
% Note: the Mathematica solution solves for the DIMENSIONLESS velocity

% define search directory
search_directory = 'Mathematica_solutions';

% find files in directory
filess = Find_Files(search_directory);
t = linspace(1E-10,50,1000);

for indexx=4:length(filess)
    files = Extract_Files(filess(indexx), filess);

    filename = strcat(search_directory, filesep, files);
    filename = char(filename);
    fid = fopen(filename);
    filedata = textscan(fid, '%f%f', 'Delimiter', ' ', 'Headerlines', 0);
    fclose(fid);
    if indexx == 4
        dim_r_p = cell2mat(filedata(:,1)); %dimensionless r_p
        dim_vel = -cell2mat(filedata(:,2)); %dimensionless velocity
    elseif indexx == 7
        dim_r_p = cell2mat(filedata(:,1)); %dimensionless r_p
        dim_vel = -cell2mat(filedata(:,2)); %dimensionless velocity
    elseif indexx == 8
        dim_r_p = cell2mat(filedata(:,1)); %dimensionless r_p
        dim_vel = -cell2mat(filedata(:,2)); %dimensionless velocity
    elseif indexx == 9
        dim_r_p = cell2mat(filedata(:,1)); %dimensionless r_p
        dim_vel = -cell2mat(filedata(:,2)); %dimensionless velocity
        dim_r_p = dim_r_p(2:2:end);
        dim_vel = dim_vel(2:2:end);
    elseif indexx == 5
        %shorten vectors TURN ON FOR DATA FILES 4 THROUGH 11
        dim_r_p = cell2mat(filedata(:,1)); %dimensionless r_p
        dim_vel = -cell2mat(filedata(:,2)); %dimensionless velocity
        dim_r_p = dim_r_p(2:2:end);
        dim_vel = dim_vel(2:2:end);
    elseif indexx == 6
        %shorten vectors TURN ON FOR DATA FILES 4 THROUGH 11
        dim_r_p = cell2mat(filedata(:,1)); %dimensionless r_p
        dim_vel = -cell2mat(filedata(:,2)); %dimensionless velocity
        dim_r_p = dim_r_p(2:2:end);
        dim_vel = dim_vel(2:2:end);
    end
end
```

```

elseif indexx == 10
    %shorten vectors TURN ON FOR DATA FILES 4 THROUGH 11
    dim_r_p = cell2mat(filedata(:,1)); %dimensionless r_p
    dim_vel = -cell2mat(filedata(:,2)); %dimensionless velocity
    dim_r_p = dim_r_p(2:2:end);
    dim_vel = dim_vel(2:2:end);
elseif indexx == 11
    %shorten vectors TURN ON FOR DATA FILES 4 THROUGH 11
    dim_r_p = cell2mat(filedata(:,1)); %dimensionless r_p
    dim_vel = -cell2mat(filedata(:,2)); %dimensionless velocity
    dim_r_p = dim_r_p(2:2:end);
    dim_vel = dim_vel(2:2:end);
elseif indexx == 12
    dim_r_p = cell2mat(filedata(:,1)); %dimensionless r_p
    dim_vel = -cell2mat(filedata(:,2)); %dimensionless velocity
    dim_r_p = dim_r_p(2:2:end);
    dim_vel = dim_vel(2:2:end);
elseif indexx ==13
    %shorten vectors TURN ON FOR DATA FILES 4 THROUGH 11
    dim_r_p = cell2mat(filedata(:,1)); %dimensionless r_p
    dim_vel = -cell2mat(filedata(:,2)); %dimensionless velocity
    dim_r_p = dim_r_p(2:2:end);
    dim_vel = dim_vel(2:2:end);
elseif indexx ==14
    %shorten vectors TURN ON FOR DATA FILES 4 THROUGH 11
    dim_r_p = cell2mat(filedata(:,1)); %dimensionless r_p
    dim_vel = -cell2mat(filedata(:,2)); %dimensionless velocity
    dim_r_p = dim_r_p(2:2:end);
    dim_vel = dim_vel(2:2:end);
end
factor = round(0.9*length(dim_vel));
dim_vel_plot(1:length(dim_vel),indexx) = dim_vel(1:length(dim_vel),1);
dim_r_p_plot(1:length(dim_r_p),indexx) = dim_r_p(1:length(dim_r_p),1);
%% Initial Conditions

% Number of nodes
nodes = length(dim_vel); %this number varies based on the Mathematica solution

% time measurements
t = linspace(1E-10,50,1000); %sec

% time increments
dt = 0.0001;

% Temperature of mold
Tb = (180-32)/1.8; %degC

% Temperature of melt
Ti = (623.6-32)/1.8; %degC

% Temperature of cooling water
Tc = 25; %degC

%Fractional coverage asymptotic value
n_p = 0.2968;
if n_p >= 0.3

```

```
m_n = 0.1516*log(n_p)+0.7259;
else
m_n = -0.1428*n_p^2+0.2626*n_p+0.4782;
end

%% Ellis model parameters

% a: (measure of degree of shear thinning behavior)
a = 3.26;

% Tao_half: (value of shear stress at which the apparent
% viscosity has dropped to half its zero shear value)
tao_half = 16618.6;

% Zero Shear Viscosity
% function of temperature
% defined later

%% Physical Properties of the tube

% External Diameter
OD = 2 * (150E-3+0.00635); %m

% Internal Diameter
ID = 2 * 0.00635; %m

% External Radius
R02 = OD * 0.5; %m

% Internal Radius
R01 = ID * 0.5; %m

% Length of tube
L = 0.585; %m

% Gravitational Acceleration
g = 9.81; %m/s^2

% Diameter
D = OD; %m

% Thermal Conductivity
k_s = 16; %W(m*K)

%Density
den_s = 7850; %kg/m^3

%Specific Heat
Cp_s = 514.5; %J/(kg*K)

% Thermal Diffusivity
a_s = k_s / (den_s * Cp_s); %m^2/s

% Step change in tube
dr2 = (R02 - R01) / (nodes - 1);
```

```

%% Physical Properties of the polymer

% Specific Heat
Cp_p = 2100; %J/(kg*K)

% Thermal Conductivity
k_p = 0.15; %W/(m*K)

% Density
den_p = 940; %kg/m^3

% Thermal Diffusivity
a_p = k_p / (den_p * Cp_p); %m^2/s

% Flow Activation Energy
%delta_H_R = 8575; %K
delta_H_R = 9.0679E3; %K

% Zero shear viscosity
zero_visc = 1*10^16*exp(-0.056*(Ti+273.13));

% Frequency factor
A = zero_visc / exp(delta_H_R / (25+273.13)); %kg/(m*s)

% Pressure gradient
delta_p = 2.42 * 10^6; %Pa

% Step change in polymer
drl = R01 / (nodes - 1);

n_drl = 1.0 / (nodes - 1);

%% Physical Properties of the cooling fluid

% Specific Heat
Cp_f = 10^-6 * Tb^4 - 0.0004 * Tb^3 + 0.0488 * Tb^2 - 2.2527 * Tb + 4212; %J/(kg*K)

% Viscosity
u_f = (4 * 10^-8 * Tb^4 - 10^-5 * Tb^3 + 0.001 * Tb^2 - 0.0557 * Tb + 1.7832) * 10^-3; %kg/(m*s)

% Density
den_f = -0.0036 * Tb^2 - 0.0656 * Tb + 1000.4; %kg/m^3

% Thermal Conductivity
k_f = -9 * 10^-6 * Tb^2 + 0.0021 * Tb + 0.5607; %W/(m*K)

% Coefficient of Volume Expansion
b = (0.0036 * 2 * Tb + 0.0656) / den_f; %1/K

% Thermal Diffusivity
a_f = k_f / (den_f * Cp_f); %m^2/s

%% Heat Transfer outside the tube wall

% Initializing temperature profiles

```

```
Temp_p = zeros(nodes,2);
Temp_s = zeros(nodes,2);
Temp_fp = zeros(nodes,length(t));
Temp_fs = zeros(nodes,length(t));

% Initializing radial position profiles
for i = nodes : -1 : 1
    r_s(i) = R01 + dr2 * (i-1);
end
for i = nodes : -1 : 1
    r_p(i) = dr1 * (i-1);
end

% Boundary Condition 1: @ t=0, T_f(r,0) = Ti
Temp_p = Temp_p + Ti;

% Boundary Condition 2: @ t=0, T_s(r,0) = Tc
Temp_s = Temp_s + Tc;

% Calculation at R01
i=nodes;
j=1;

% Grashof Number
Gr = D^3 * den_f^2 * g * b * (Temp_s(i,j)-Tb) / u_f^2;

% Prandtl Number
Pr = Cp_f * u_f / k_f;

% Raleigh Number
Ra = Gr * Pr;

% Reynolds Number
Re = 1;

% Nusselt Number
Nu = 0.62*Re^(1/2)*Pr^(1/3)/[1+(0.4/Pr)^(2/3)]^(1/4);

% Heat Transfer Coefficient
h = k_f * Nu / D;

%Initialization of counter variables
count = 0;

for k = 1 : length(t)

    disp(length(t)-k+1)

    while count <= t(k)

        %counter variables incrementation
        count = count + dt;

        % Boundary Condition 3: @ r=R02, -ks*dT/dr = h(Ts(R02,t)-Tb)
        i=nodes;
```

```

j=1;
Temp_s(i,j+1) = Temp_s(i,j) + (2*a_s*dt/dr2^2)*(Temp_s(i-1,j)-Temp_s(i,j)-((R02+
(dr2/2))/R02)*(h/k_s)*dr2*(Temp_s(i,j)-Tb));

% Temperature profile in Solid
for i = nodes-1 : -1 : 2

    Temp_s(i,j+1) = Temp_s(i,j) + (a_s*dt/(r_s(i)*dr2^2))*(((r_s(i)+r_s(i-1))/2)
*Temp_s(i-1,j)-2*r_s(i)*Temp_s(i,j)+((r_s(i+1)+r_s(i))/2)*Temp_s(i+1,j));

end

% Boundary Condition 4: @ r=R01, solid-liquid interface energy balance
i = 1;
Temp_s(i,j+1) = Temp_s(i,j) + (2*k_s*dt*(R01+dr2/2)*(Temp_s(i+1,j)-Temp_s(i,j))/(dr2*
((R01+dr2/4)*den_s*Cp_s*dr2+(R01-dr1/4)*(-0.58 * Temp_p(nodes,j) + 903)*Cp_p*dr1)))+(2*k_p*dt*
(R01-dr1/2)*(Temp_p(nodes-i,j)-Temp_s(i,j))/(dr1*((R01+dr2/4)*den_s*Cp_s*dr2+(R01-dr1/4)*(-0.58 *
Temp_p(nodes,j) + 903)*Cp_p*dr1)));

Temp_p(nodes,j+1) = Temp_s(i,j+1);

% Temperature profile in Polymer
for i = nodes-1 : -1 : 2

    Temp_p(i,j+1) = Temp_p(i,j) + ((k_p/((-0.58 * Temp_p(i,j) + 903)*Cp_p))*dt/(r_p
(i)*dr1^2))*(((r_p(i)+r_p(i-1))/2)*Temp_p(i-1,j)-2*r_p(i)*Temp_p(i,j)+((r_p(i+1)+r_p(i))/2)
*Temp_p(i+1,j));

end

% Boundary Condition 5: @r=0, kdT/dr = 0
i=1;
Temp_p(i,j+1) = Temp_p(i,j) + (4*(k_p/((-0.58 * Temp_p(i,j) + 903)*Cp_p))*dt/dr1^2)*
(Temp_p(i+1,j)-Temp_p(i,j));

for i = 1 : nodes

    Temp_s(i,j) = Temp_s(i,j+1);
    Temp_p(i,j) = Temp_p(i,j+1);

end

end

% Creating Temperature Profiles for Polymer and Solid
for i = 1 : nodes

    Temp_fp(i,k) = Temp_p(i,j);
    Temp_fs(i,k) = Temp_s(i,j);

end

end

%% Temperature Profile Calculation
% figure
% plot(r_p,Temp_fp(1:nodes,4))

```

```
% hold on
% plot(r_p,Temp_fp(1:nodes,5))
% plot(r_p,Temp_fp(1:nodes,7))

%% Velocity Profile Calculation

%initialize eta_o
eta_o = zeros(nodes,length(t));
%calculate eta_o
for k = 1 : length(t)
    for i = 1 : nodes
        eta_o(i,k) = 1*10^16*exp(-0.056*(Temp_fp(i,k)+273.13));
    end
end

% THE INPUT OF THE MATHEMATICA SOLUTION IS IMPORTANT FOR THE FOLLOWING
% PORTION OF CODE

%plot dimensionless velocity profile
figure
plot(dim_vel, dim_r_p*0.00635);
hold on
plot(dim_vel, -dim_r_p*0.00635)
ylabel('Radius')
xlabel('Dimensionless Velocity')
title('Dimensionless Velocity Profile for Ellis Model for PS')
axis([0 1 -0.00635 0.00635])

% solving for normalized velocity... eventually
% but first, non-normalized velocity,
% initialize velocity matrix
vel = zeros(nodes,length(t));
% velocity profile calculation
for k = 1 : length(t)
    for i = nodes-1 : -1 : 1

        vel(i,k) = dim_vel(i)*delta_p*R01^2/(L*eta_o(i,k));

    end
end

% Plot Velocity Profile
figure
hold on
plot(vel(1:nodes,1),dim_r_p)
plot(vel(1:nodes,10),dim_r_p)
plot(vel(1:nodes,20),dim_r_p)
plot(vel(1:nodes,30),dim_r_p)
plot(vel(1:nodes,40),dim_r_p)
plot(vel(1:nodes,50),dim_r_p)
plot(vel(1:nodes,60),dim_r_p)
plot(vel(1:nodes,70),dim_r_p)
plot(vel(1:nodes,80),dim_r_p)
plot(vel(1:nodes,90),dim_r_p)
plot(vel(1:nodes,100),dim_r_p)
```

```

plot(vel(1:nodes,1),-dim_r_p)
plot(vel(1:nodes,10),-dim_r_p)
plot(vel(1:nodes,20),-dim_r_p)
plot(vel(1:nodes,30),-dim_r_p)
plot(vel(1:nodes,40),-dim_r_p)
plot(vel(1:nodes,50),-dim_r_p)
plot(vel(1:nodes,60),-dim_r_p)
plot(vel(1:nodes,70),-dim_r_p)
plot(vel(1:nodes,80),-dim_r_p)
plot(vel(1:nodes,90),-dim_r_p)
plot(vel(1:nodes,100),-dim_r_p)
ylabel(' Normalized Radius')
xlabel('Velocity')
title('Velocity Profile for CE Model for PS')

% Now that we can calculate vel_max we can find normaliized velocity!
% normalizing the velocity profile
n_vel = zeros(nodes,length(t));

for k = 1 : length(t)

    for i = 1 : nodes

        n_vel(i,k) = vel(i,k) / vel(1,k);

    end

end

% Plot Normalized Velocity Profile
figure
hold on
plot(n_vel(1:nodes,10),dim_r_p)
%plot(n_vel(1:157,10),dim_r_p)
plot(n_vel(1:nodes,200),dim_r_p)
%plot(n_vel(1:157,30),dim_r_p)
plot(n_vel(1:nodes,400),dim_r_p)
%plot(n_vel(1:157,50),dim_r_p)
plot(n_vel(1:nodes,600),dim_r_p)
%plot(n_vel(1:157,70),dim_r_p)
plot(n_vel(1:nodes,800),dim_r_p)
%plot(n_vel(1:157,90),dim_r_p)
plot(n_vel(1:nodes,1000),dim_r_p)
plot(n_vel(1:nodes,10),-dim_r_p)
%plot(n_vel(1:157,10),-dim_r_p)
plot(n_vel(1:nodes,200),-dim_r_p)
%plot(n_vel(1:157,30),-dim_r_p)
plot(n_vel(1:nodes,400),-dim_r_p)
%plot(n_vel(1:157,50),-dim_r_p)
plot(n_vel(1:nodes,600),-dim_r_p)
%plot(n_vel(1:157,70),-dim_r_p)
plot(n_vel(1:nodes,800),-dim_r_p)
%plot(n_vel(1:157,90),-dim_r_p)
plot(n_vel(1:nodes,100),-dim_r_p)
ylabel('Normalized Radius')
xlabel('Normalized Velocity')

```



```

title('Normalized Velocity Profile for Ellis Model for PS')
box on
legend('t=1 sec','t=20 sec','t=40 sec','t=60 sec','t=80 sec','t=100 sec')

%%

for g = 1:1000
    clear r_p
    for i = nodes : -1 : 1
        r_p(i) = drl * (i-1);
    end
    Qm=trapz(r_p,vel(1:nodes,g));
    Q_m(indexx,g)=Qm;
end
end
%% Plot All Dimensionless Velocity Profiles
figure
plot(dim_vel_plot(1:173,4),dim_r_p_plot(1:173,4))
hold on
plot(dim_vel_plot(1:173,4),-dim_r_p_plot(1:173,4))
% title('\tau_{wall}=1.47E6 Pa')
ylabel('dimensionless radius')
xlabel('dimensionless velocity')
for i = 5:length(filess)
    plot(dim_vel_plot(1:173,i),dim_r_p_plot(1:173,i))
end
for i = 5:length(filess)
    plot(dim_vel_plot(1:173,i),-dim_r_p_plot(1:173,i))
end
title('All Dimensionless Velocity Profiles')

%% Fractional Coverage Calculation

%initialize vectors
m = zeros(length(t),1);
Rx = zeros(length(t),1);

%define shear stress average and wall
tau_avg = linspace(1.47E6,3.42,length(t));
tau_wall = tau_avg(1); %isothermal

% Rx calculation
for k = 1 : length(t)

    Rx(k) = R01*sqrt(tau_avg(k)*eta_o(1,k)/(tau_wall*eta_o(105,k)));

    %Fractional coverage when integrating through ru*
    m(k) = 1 - (1 - m_n) * (Rx(k) / R01)^2;

end

%% Experimental Data

Fo_exp = [0.009290161    0.009361771 0.009319921 0.00923157    0.00912741    0.00878424 ✓
0.008636369 0.008528489 0.007097774 0.006588133 0.005670779 0.00480439    0.004039928 0    0    0 ✓

```

```

0.010646289 0.01123963 0.011360531 0.011388431 0.011343791 0.01123963 0.01102573 0.01085275 ✓
0.010718829 0.010227788 0.010034348 0.009548658 0.009240126 0.008843946 0.005331866 0.001957565 ✓
0.001186136 0.000718709 0.000435483 0.009763718 0.010065503 0.010091078 0.010046903 0.009961343 ✓
0.009831143 0.009605617 0.009436822 0.009310342 0.00869282 0.008356928 0.008006014 0.007669818 ✓
0.001254455 0.000748789 0.000446954 0.000266789 0.013721433 0.014074369 0.014123194 0.014102269 ✓
0.014035309 0.013923709 0.013566588 0.013449408 0.013030907 0.012869087 0.012549687 0.01221483 ✓
0.011892585 0.000900184 0.000436854 0.000212003 0.000102884 1.90411E-05 0.007187798 0.007228718 ✓
0.007172918 0.007070618 0.006955298 0.006810218 0.006430777 0.006317317 0.005789076 0.00551253 ✓
0.005229214 0.004961374 0.000574741 0 0 0.009706057 0.010012958 0.010078058 0.010096658 ✓
0.010078058 0.010029698 0.009843698 0.009778598 0.009538657 0.0092543 0.00905196 0.008857896 ✓
0.001638369 0.000986789 0.000594343 0.000357973 0.010386076 0.01071855 0.010758076 0.01072785 ✓
0.01065345 0.01053441 0.010322835 0.010162409 0.009446308 0.009120212 0.008779032 0.008451206 ✓
0.000800971 0.00035954 0.00016139 7.24447E-05 0.010082522 0.010394538 0.010424763 0.010385238 ✓
0.010303398 0.010176917 0.009790037 0.009665416 0.009055335 0.008722543 0.008375039 0.008041633 ✓
0.002343016 0 0.009265981 0.009445006 0.009414781 0.009314806 0.009184605 0.009009765 ✓
0.008526164 0.008377364 0.007670562 0.007294246 0.006905696 0.00653596 0.000510563 0.000135167 ✓
6.95478E-05 0.009419617 0.009409387 0.009330337 0.009204786 0.009070866 0.008907186 0.008490545 ✓
0.008367785 0.007802344 0.007509058 0.007209932 0.006928142 0.001532225 0.000979925 0.000626705 ✓
0.000400805 0.000256333 0.009973712 0.010556823 0.010673073 0.010696323 0.010647963 0.010540083 ✓
0.010321533 0.010145762 0.010009982 0.009513361 0.009318061 0.008928142 0.008517329 0.008118358 ✓
0.000379347 0.000187439 9.26151E-05 1.78759E-05];
m_exp = [0.664455133 0.660600195 0.659903584 0.653815611 0.643863879 0.643420578 0.6420786 ✓
0.641032095 0.63 0.6320329 0.630065371 0.6241699 0.61863233 0.527927299 0.511768978 ✓
0.498364331 0.689057396 0.692621509 0.692304866 0.689900398 0.68720811 0.68546886 0.685555 ✓
0.684666754 0.687869965 0.680615113 0.688518593 0.669690715 0.667162285 0.66521013 0.643249142 ✓
0.562980759 0.533264801 0.512741854 0.505587609 0.672333745 0.680039601 0.667376696 0.668113545 ✓
0.669887703 0.665519991 0.66042195 0.663392543 0.65819495 0.651606294 0.642900574 0.641887469 ✓
0.635561537 0.536369509 0.520796145 0.51029797 0.492728249 0.71303761 0.6994871 0.701769188 ✓
0.69999603 0.701257595 0.698408593 0.699423999 0.698485672 0.699962583 0.699467845 0.686372017 ✓
0.680490058 0.670616312 0.54996579 0.516916765 0.498031168 0.493316219 0.486415845 0.646852772 ✓
0.641527395 0.6402977 0.642550577 0.641688346 0.640829917 0.634794885 0.629447227 0.622278177 ✓
0.619749266 0.613333547 0.602327828 0.541584269 0.527157593 0.518919719 0.675408624 0.668449729 ✓
0.665198697 0.666624644 0.661579087 0.658701847 0.659627084 0.658785575 0.65831585 0.648579795 ✓
0.642704076 0.639065328 0.568488481 0.537996416 0.522383791 0.512449143 0.698873255 0.692935765 ✓
0.690407814 0.691673269 0.688984876 0.692774128 0.694173317 0.688442735 0.678472635 0.669053441 ✓
0.661023285 0.658290052 0.586277093 0.526014283 0.515202825 0.496022157 0.679814469 0.678385666 ✓
0.678443498 0.675245682 0.675144517 0.678017961 0.670001147 0.671366624 0.669771592 0.659192099 ✓
0.656571262 0.650316108 0.593335238 0.547726996 0.673297245 0.672809622 0.669223792 0.668933217 ✓
0.667955017 0.66403757 0.662242917 0.659374749 0.657487586 0.654273822 0.636189927 0.625708532 ✓
0.500466487 0.473828363 0.457995038 0.693084074 0.690318652 0.687879622 0.68319826 0.684136001 ✓
0.679103836 0.679561568 0.67289133 0.6701 0.66521 0.65222 0.6294 0.575210798 0.539712115 ✓
0.529919918 0.527618377 0.515450591 0.686706115 0.679323206 0.684293066 0.689236851 0.684060921 ✓
0.686483024 0.681287794 0.677176137 0.672125212 0.668292035 0.667041945 0.665278969 0.661918419 ✓
0.65875516 0.562243403 0.544177804 0.532056223 0.511946383];

```

```
%% Fourier Number
```

```
Fo = a_p*t/R01^2;
```

```
%% Plot Fractional Coverage vs Fourier Number
figure
```

```
semilogx(Fo, m, Fo_exp, m_exp, 'rs')
```

```
xlabel('Fo')
```

```

ylabel('m')

title('Polystyrene Ellis Model')

legend('Simulated Data','Experimental Data')

axis([3E-5 3E-2 0.4 0.9])

%%
close all
%% Determine simulated vs experimental data: determination of \tau_{wall}
dp=linspace(1,181,181);
for dp=1:length(dp)
    disp(dp)
convertedrestime = xlsread('PS_Fractional_Cov_Comp.xlsx','F2:F182');
convertedrowlength = xlsread('PS_Fractional_Cov_Comp.xlsx','G2:G182');
Q_exp = xlsread('PS_Fractional_Cov_Comp.xlsx','E2:E182');
for i = 1:length(convertedrestime)
    convertedrestime(i) = round(convertedrestime(i));
    convertedrowlength(i) = round(convertedrowlength(i));
end
if convertedrestime(dp) == 0
    disp('Error')
else
    tau = linspace(1.168E5,1.47E6,8);
    for i = 1:8
        Q_e(i)=Q_exp(dp);
    end
    Q_m_ref=Q_m(7:length(filess),convertedrestime(dp));
    [tau_wall_int(dp),Q_e_int(dp)]=intersections(tau,Q_m_ref,tau,Q_e);
    % figure
    % loglog(tau,Q_m_ref)
    % hold on
    % loglog(tau,Q_e)
    % xlabel('\tau_{wall} (Pa-s^2)')
    % ylabel('Q (m/s)')
end
end

%% Determine simulated vs experimental data: solve for Rx
%tau_wall_int range = [3E5, 6.6E5]
% Based on solved dimensionless velocity profiles estimate tau_wal~3E5
indexx=13;
if indexx == 13
    disp('i')
    files = Extract_Files(filess(indexx), filess);

    filename = strcat(search_directory, filesep, files);
    filename = char(filename);
    fid = fopen(filename);
    filedata = textscan(fid, '%f%f', 'Delimiter', ' ', 'Headerlines', 0);
    fclose(fid);
    dim_r_p = cell2mat(filedata(:,1)); %dimensionless r_p
    dim_vel = -cell2mat(filedata(:,2)); %dimensionless velocity

    %initialize eta_o

```

```
eta_o = zeros(nodes,length(t));
%calculate eta_o
for k = 1 : length(t)
    for i = 1 : nodes
        eta_o(i,k) = 1*10^16*exp(-0.056*(Temp_fp(i,k)+273.13));
    end
end

% initialize velocity matrix
vel = zeros(nodes,length(t));
% velocity profile calculation
for k = 1 : length(t)
    for i = nodes-1 : -1 : 1

        vel(i,k) = dim_vel(i)*delta_p*R01^2/(L*eta_o(i,k));

    end

end

n_vel = zeros(nodes,length(t));

for k = 1 : length(t)

    for i = 1 : nodes

        n_vel(i,k) = vel(i,k) / vel(1,k);

    end

end

% Fractional Coverage Calculation

%initialize vectors
m = zeros(length(t),1);
Rx = zeros(length(t),1);

%define shear stress average and wall
tau_avg = linspace(1.47E6,3.42,length(t));
tau_wall = tau_avg(1); %isothermal

% Rx calculation
for k = 1 : length(t)

    Rx(k) = R01*sqrt(tau_avg(k)*eta_o(1,k)/(tau_wall*eta_o(105,k)));

    %Fractional coverage when integrating through ru*
    m(k) = 1 - (1 - m_n) * (Rx(k) / R01)^2;

end
end

%% Determine simulated vs experimental data: determination of viscosity

% determine m_sim
dp=linspace(1,181,181);
for i=1:length(dp)
```

```

m_sim = zeros(length(convertedrestime),1);

for i = 1:length(convertedrestime)
    if convertedrestime(i) == 0
        disp('Error')

        elseif convertedrestime(i) < 1001
            m_sim(i) = m(convertedrestime(i));
        else
            m_sim(i) = 0;
        end
    end
end

% calculate vel_sim
vel_sim = zeros(length(convertedrowlength),1);
for i = 1:length(convertedrowlength)
    if convertedrestime(i) == 0
        disp('Error')
    else
        vel_sim(i,i) = vel(convertedrowlength(i),convertedrestime(i));
    end
end

% calculate shear rate (gamma_dot)
vel_diff = zeros(length(vel_sim),1);
t_diff = zeros(length(vel_sim),1);
gamma_dot = zeros(length(vel_sim),1);
for i = 1:length(vel_sim)
    if convertedrestime(i) == 0
        disp('Error')
    else
        vel_diff(i) = vel(convertedrowlength(i)-1,convertedrestime(i))-vel(convertedrowlength(i),
convertedrestime(i));
        t_diff(i) = 0.05;
        gamma_dot(i) = vel_diff(i)/t_diff(i);
    end
end

%%
% piston speed data references
% data_points = [2 37 18 9 45 26 50 33 53 36];

% delay time
% data_points = [54 37 72 61 45 79 67 50 84 70 53];

%gas pressure
% data_points = [87 37 103 94 45 110 99 50 115 102 53 118];

% shot size
data_points = [119 37 133 126 45 140 131 50 145 53 147];

% melt temperature
% data_points = [148 37 165 155 45 173 160 50 163 53 181];

Ti_dp = xlsread('PS_Fractional_Cov_Comp.xlsx','I2:I182'); %degC

```

```

for data_point_no = 1:length(data_points)
    dp = data_points(data_point_no);
    % plot viscosity
    lenn = 1E2;
    Ti = Ti+273.15; %K
    m_power = A.*exp(delta_H_R./Ti); %Pa-s
    shear_rate = logspace(-3, 10, lenn);
    n_p = 0.2968;
    B = 1.6783E-6;
    Tb = 1.1376E4;
    no = B.*exp(Tb./Ti_dp(data_points(data_point_no)));
    no_exp = B*exp(Tb/Ti_dp(data_points(data_point_no)));
    tau_star = 1.738E4;
    for k = 1:length(shear_rate)
        for i = 1:length(Ti)
            viscosity(i,k) = no(i)/(1+(no(i)*shear_rate(k)/tau_star)^(1-n_p));
            visc_dp(data_point_no,k) = viscosity(i,k);
        end
    end
    figure (3)
    loglog(shear_rate, viscosity(1,1:lenn))
    hold on

    % plot data points
    for i = 1:length(gamma_dot)
        visc_exp(i) = no_exp/(1+(no_exp*gamma_dot(i)/tau_star)^(1-n_p));
    end
    visc_exp_dp(data_point_no) = visc_exp(data_point_no);
    loglog(gamma_dot(1), visc_exp(1), 'rs')
    leg_ent = ['ABCDEFGHIJKLMN'];
    legend(leg_ent(1))
    xlabel('\gamma_{dot} [1/s]')
    ylabel('\eta [Pa-s]')
    hold off

    % Plot m_sim vs m_exp
    figure (4)
    m_exp_dp(data_point_no) = [m_exp(data_points(data_point_no)-1)];
    m_sim_dp(data_point_no) = m_sim(data_points(data_point_no)-1);
    plot(m_sim_dp(data_point_no), m_exp_dp(data_point_no), 'rs')
    xlabel('m_{sim}')
    ylabel('m_{exp}')
    leg_ent = ['ABCDEFGHIJKLMNABCDEFGHIJKLMNABCDEFGHIJKLMNABCDEFGHIJKLMNABCDEFGHIJKLMNABCDEFGHIJKLMNABCDEFGHIJKLMN'];
    legend(leg_ent(data_point_no))
    axis([0.5 1 0.5 1])
    hold on
    line = linspace(0.5,1,100);
    plot(line,line)
end
%% Define Data Points

visc_exp_dpp=visc_exp_dp;
clear visc_exp_dp
for i = 1:length(data_points)

```

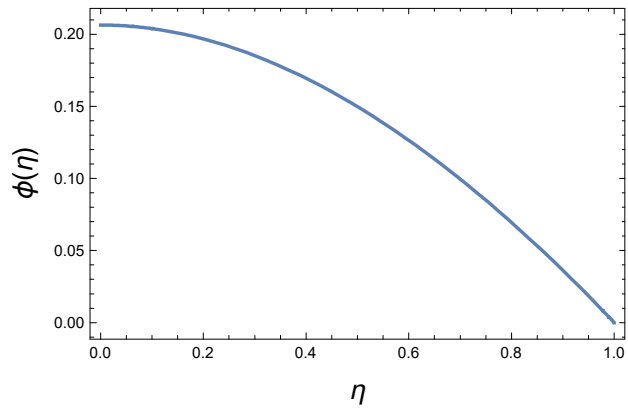
[illegible]

15 Appendix H: Mathematica code for PS: Cross-Exponential Model

```

ClearAll["Global`*"]
R = 0.00635; (*m*)
delPL = 2420000; (*Pa*)
T = {358.209114772210, 358.209114772210, 358.209114772210, 358.209114772210,
58.209114780497, 358.209115153781, 358.209121591472, 358.209179879124,
358.209515036587, 358.210896401797, 358.215331208706, 358.227068418781,
358.253776265434, 358.307660841144, 358.406235985968, 358.572807581777,
358.836211202275, 359.230514067271, 359.794243069220, 360.568981457907,
361.599279718692, 362.930476581638, 364.609240053483, 366.682680892601,
369.196418805698, 372.197481216343, 375.730237013768, 379.840434898080,
384.574157656625, 389.974803415770, 396.090373508991, 402.965284687088,
410.648969074409, 419.193437607716, 428.647966474025, 439.071922380453,
450.520214711697, 463.057811129546, 476.755034731154, 491.678976970746,
507.913687369223, 525.536983450300, 544.642942462401, 565.334186087718,
587.708893295345, 611.891068945741, 637.995652101931, 666.161982544949,
696.542081489387, 729.281613359925, 764.564208566193, 802.560353591851,
843.484644461334, 887.536459767833, 934.956602799835, 986.007306519566,
1040.94028710230, 1100.07114484117, 1163.69356483919, 1232.16115143828,
1305.85849137154, 1385.15502120982, 1470.51239551765, 1562.36058630588,
1661.21596287046, 1767.63947149223, 1882.16993489342, 2005.47902327938,
2138.19219486422, 2281.05914656413, 2434.89328904772, 2600.47514789544,
2778.77620125508, 2970.69979204574, 3177.32733873120, 3399.83056114398,
3639.33143219877, 3897.22558063904, 4174.80715214072, 4473.62448406337,
4795.35262066395, 5141.59027996188, 5514.32682289482, 5915.39927103894,
6347.00565279730, 6811.51976894279, 7311.19725328897, 7848.84866226162,
8427.05481562460, 9048.90618660127, 9717.73402951443, 10436.6860165114,
11209.6935595937, 12040.3412304040, 12932.9282520195, 13892.0792025874,
14922.1339080227, 16028.5318429920, 17216.1900243504, 18491.0201498321}; (*deg C*)
mu0 = 1.6783 * 10^-6 * Exp[1.1376 * 10^4 / T]; (*NOT CONSTANT*) (*Pa-s*)
vv = (delPL) R^2 / mu0; (*NOT CONSTANT*) (*1/s*)
tau = mu0 * vv / (1.738 * 10^4 * R); (*CONSTANT*) (*dimensionless*)
n = 0.2968; tau = 0.884177; (*BOTH CONSTANT*)
system[Omega_] :=
{phi'[eta] == psi[eta], D[eta] (eta (1 + (tau psi[eta]))^(n-1) psi[eta]) == -eta, phi[0.0001] == Omega, psi[0.0001] == 0};
myODEsoln[Omega_?NumericQ] := NDSolve[system[Omega], {psi[eta], phi[eta]}, {eta, 0.0001, 1}];
endpoint[Omega_?NumericQ] := First[phi[eta] /. myODEsoln[Omega] /. eta -> 1]
bc = FindRoot[endpoint[Omega], {Omega, -2, 2}];
Plot[Evaluate[phi[eta] /. myODEsoln[Omega /. bc]], {eta, 0.0001, 1}, PlotStyle -> Thick,
Frame -> True, FrameLabel -> {Style["eta", 16], Style["phi(eta)", 16]}]
(*Extraction of Data from Plot*)
data = Cases[Plot[Evaluate[phi[eta] /. myODEsoln[Omega /. bc]], {eta, 0.0001, 1}],
Line[data_] -> data, -4, 1][[1]];
Export["CE_Model_PS_data.txt", data, "Table"]

```

CE_Model_PS_data.txt

```
 $\phi 1 = \text{First}[\phi[\eta] /. \text{myODEsoln}[\Omega /. \text{bc}]];$   
 $Q = \text{NIntegrate}[2 \pi \eta \phi 1, \{\eta, 0.0001, 1\}]$ 
```

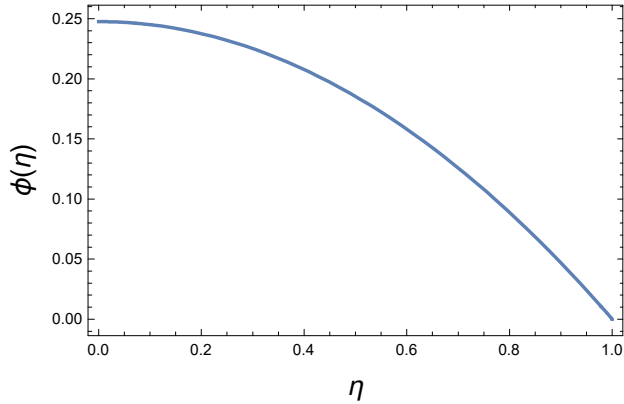
0.312047

```

Rx = 0.0003;
delPL = 2420000;
T = {252};
 $\mu_0 = 1.6783 \times 10^{-6} \cdot \text{Exp}[1.1376 \times 10^4 / T]$  (*NOT CONSTANT*)
vv = (delPL) Rx2 /  $\mu_0$ ; (*NOT CONSTANT*)
 $\tau = \mu_0 \cdot vv / (1.738 \times 10^4 \cdot Rx)$  (*CONSTANT*)
n = 0.2968;  $\tau = 0.04177215189873417$ ; (*BOTH CONSTANT*)
(*BECAUSE BOTH PARAMETERS ARE NOT FUNCTIONS OF TEMPERATURE FOR THIS NORMALIZED SOLUTION,
TEMPERATURE DEPENDENCE WOULD ONLY APPEAR IN PLOTS OF R VS UZ CORRECT? SINCE UZ=
phi*vv and vv is a function of temperature*)
system[ $\Omega$ ] := { $\phi'[\eta] = \psi[\eta]$ ,  $\partial_\eta (\eta (1 + (\tau \psi[\eta]))^{n-1} \psi[\eta]) = -\eta$ ,
 $\phi[0.0001] = \Omega$ ,  $\psi[0.0001] = 0$ };
myODEsoln[ $\Omega$ ?NumericQ] := NDSolve[system[ $\Omega$ ], { $\psi[\eta]$ ,  $\phi[\eta]$ }, { $\eta$ , 0.0001, 1}];
endpoint[ $\Omega$ ?NumericQ] := First[ $\phi[\eta]$  /. myODEsoln[ $\Omega$ ] /.  $\eta \rightarrow 1$ ]
bc = FindRoot[endpoint[ $\Omega$ ], { $\Omega$ , -2, 2}];
Plot[Evaluate[ $\phi[\eta]$  /. myODEsoln[ $\Omega$  /. bc]], { $\eta$ , 0.0001, 1}, PlotStyle -> Thick,
Frame -> True, FrameLabel -> {Style[" $\eta$ ", 16], Style[" $\phi(\eta)$ ", 16]}}
 $\phi1 = \text{First}[\phi[\eta] /. \text{myODEsoln}[\Omega /. bc]]$ ;
Q = NIntegrate[ $2 \pi \eta \phi1$ , { $\eta$ , 0.0001, 1}]
{ $6.76337 \times 10^{13}$ }

{0.0417722}

```



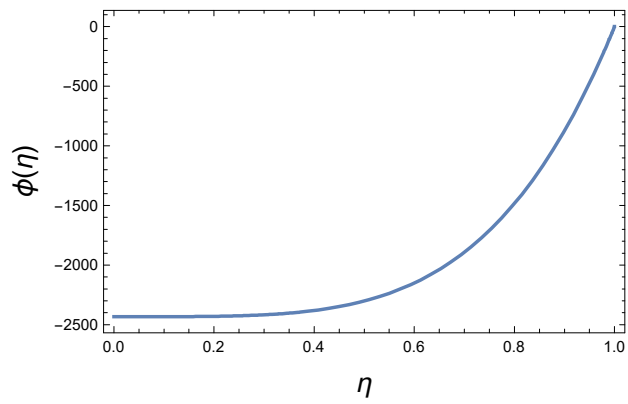
0.388129

16 Appendix I: Mathematica code for PS: Ellis Model

```

ClearAll["Global`*"]
R = 0.00635; (*m*)
delPL = 2420000; (*Pa/m*)
(*NOTE: PARAMETERS ARE FOR PS*)
T = {358.209114772210, 358.209114772210, 358.209114772210, 358.209114772210,
58.209114780497, 358.209115153781, 358.209121591472, 358.209179879124,
358.209515036587, 358.210896401797, 358.215331208706, 358.227068418781,
358.253776265434, 358.307660841144, 358.406235985968, 358.572807581777,
358.836211202275, 359.230514067271, 359.794243069220, 360.568981457907,
361.599279718692, 362.930476581638, 364.609240053483, 366.682680892601,
369.196418805698, 372.197481216343, 375.730237013768, 379.840434898080,
384.574157656625, 389.974803415770, 396.090373508991, 402.965284687088,
410.648969074409, 419.193437607716, 428.647966474025, 439.071922380453,
450.520214711697, 463.057811129546, 476.755034731154, 491.678976970746,
507.913687369223, 525.536983450300, 544.642942462401, 565.334186087718,
587.708893295345, 611.891068945741, 637.995652101931, 666.161982544949,
696.542081489387, 729.281613359925, 764.564208566193, 802.560353591851,
843.484644461334, 887.536459767833, 934.956602799835, 986.007306519566,
1040.94028710230, 1100.07114484117, 1163.69356483919, 1232.16115143828,
1305.85849137154, 1385.15502120982, 1470.51239551765, 1562.36058630588,
1661.21596287046, 1767.63947149223, 1882.16993489342, 2005.47902327938,
2138.19219486422, 2281.05914656413, 2434.89328904772, 2600.47514789544,
2778.77620125508, 2970.69979204574, 3177.32733873120, 3399.83056114398,
3639.33143219877, 3897.22558063904, 4174.80715214072, 4473.62448406337,
4795.35262066395, 5141.59027996188, 5514.32682289482, 5915.39927103894,
6347.00565279730, 6811.51976894279, 7311.19725328897, 7848.84866226162,
8427.05481562460, 9048.90618660127, 9717.73402951443, 10436.6860165114,
11209.6935595937, 12040.3412304040, 12932.9282520195, 13892.0792025874,
14922.1339080227, 16028.5318429920, 17216.1900243504, 18491.0201498321};
 $\mu_0 = 2 \times 10^8$ ; (* $1.6783 \times 10^{-6} \text{Exp}[1.1376 \times 10^4/T]$ *) (*NOT CONSTANT*)
vv = (delPL) R2 /  $\mu_0$ ; (*NOT CONSTANT*)
n = 3.26;  $\tau = 1.329 \times 10^6 / 16618.6$ ; (*BOTH CONSTANT*)
system[ $\Omega$ ] :=
{ $\phi'[\eta] = \psi[\eta]$ ,  $\partial_\eta \left( \eta \left( (1 + (\tau \eta))^{n-1} \right)^{-1} \psi[\eta] \right) = \eta$ ,  $\phi[0.0001] = \Omega$ ,  $\psi[0.0001] = 0$ };
myODEsoln[ $\Omega$ ?NumericQ] := NDSolve[system[ $\Omega$ ], { $\psi[\eta]$ ,  $\phi[\eta]$ }, { $\eta$ , 0.0001, 1}];
endpoint[ $\Omega$ ?NumericQ] := First[ $\phi[\eta]$  /. myODEsoln[ $\Omega$ ] /.  $\eta \rightarrow 1$ ]
bc = FindRoot[endpoint[ $\Omega$ ], { $\Omega$ , -2, 2}];
Plot[Evaluate[ $\phi[\eta]$  /. myODEsoln[ $\Omega$  /. bc]], { $\eta$ , 0.0001, 1}, PlotStyle -> Thick,
Frame -> True, FrameLabel -> {Style[" $\eta$ ", 16], Style[" $\phi(\eta)$ ", 16]}]
(*Extraction of Data from Plot*)
data = Cases[Plot[Evaluate[ $\phi[\eta]$  /. myODEsoln[ $\Omega$  /. bc]], { $\eta$ , 0.0001, 1}],
Line[data_] -> data, -4, 1][[1]];
Export["Ellis_Model_PS_data_wallstress10.txt", data, "Table"]

```



Ellis_Model_PS_data_wallstress10.txt

17 Appendix J: Data for PC

Fourier and Capillary number analysis for the polycarbonate data from the second set of experiments									
Thermal Diffusivity (m ² /s)			0.00000153						
n			0.03722						
Tb (s)			860.2						
T (s)			942.0						
h (Pa.s)			0.00010242						
R (s)			0.05935						
High delay time setting			Rowlength (mm)	Residual Time (sec)	Fourier #	Bubble velocity (m/sec)	Solid layer fractional coverage	Adjusted Radius (m)	
			90	6.3404	0.096232248	0.37871583	0.084989024	0.006074169	
			145	6.11295	0.09282545	0.35661591	0.067052486	0.006064349	
			170	5.9964	0.091051143	0.402083317	0.08887281	0.006061208	
			195	5.86265	0.088981135	0.453137674	0.096264459	0.006030561	
			215	5.7665	0.0872210528	0.499021501	0.094312915	0.006043489	
			235	5.62265	0.085338063	0.54910801	0.094321765	0.006043393	
			260	5.4564	0.082813222	0.619343002	0.091321760	0.006040786	
			275	5.35005	0.081120188	0.665580132	0.093165485	0.006040869	
			295	5.27281	0.080002223	0.695027461	0.09514279	0.006035863	
			315	5.20585	0.07755668	0.846118417	0.095517713	0.006079727	
			343	4.81481	0.0730774896	0.922743122	0.096101097	0.006071164	
			360	4.6664	#	1.000948451	0.097040442	0.006031020	
			375	4.53055	0.068764511	1.224041384	0.098019329	0.006036097	
			403	4.26531	0.064715781	2.471832128	0.096071171	0.00617267	
			415	4.14665	0.06290621	3.348610721	0.095401002	0.006043903	
			422	4.07816	0.061866465	3.982100343	0.095091323	0.006040745	
			445	3.2845	0.059067205	7.091563486	0.095767034	0.006071885	
			460	2.519	0.058252491	10.33623750	0.074570815	0.006108651	
			475	1.7435	0.054642199	15.06176821	0.061847263	0.006150501	
			490	0.868	0.014401946	21.84784025	0.047020775	0.006189051	
			505	0.1925	0.002921694	31.08154645	0.030134064	0.00625159	
			525	-	-	52.8342489	0.021179	0.00620015	
			90	5.4356	0.082409578	0.37925424	0.075454356	0.006109175	
			145	5.81175	0.078647366	0.345103097	0.076602478	0.006101342	
			170	5.0484	0.076621249	0.331101038	0.077172481	0.006106116	
			195	4.90373	0.0744217684	0.43574866	0.082404974	0.006091391	
			215	4.77993	0.072348804	0.463014586	0.080662251	0.006088113	
			240	4.64893	0.070504062	0.503611627	0.080739316	0.006081192	
			260	4.47511	0.067921414	0.55948003	0.080307053	0.006050688	
			275	4.36337	0.065620208	0.95781045	0.08063033	0.006081172	
			285	4.28975	0.06511857	0.621171918	0.079944185	0.006096825	
			315	4.052975	0.061514556	0.704711278	0.08450179	0.00607786	
			325	3.91915	0.060200034	0.74991217	0.08360524	0.006176717	
			343	3.817159	0.057952420	0.792700851	0.081548013	0.006073916	
			360	3.6671	0.055627888	0.851368485	0.087273969	0.006066562	
			375	3.53035	0.053942717	1.12678796	0.086603052	0.006068088	
			403	3.354519	0.049544627	1.077277339	0.083470827	0.006072907	
			415	3.145975	0.047718841	2.51662594	0.08241347	0.006082172	
			422	3.075744	0.046682505	2.98835972	0.081710066	0.006085013	
			445	2.4465	0.03971227	4.29948095	0.07534486	0.00613486	
			460	2.021	0.030677991	6.21781838	0.066601132	0.006134899	
			475	1.4255	0.021657122	8.40776077	0.055156686	0.006168477	
			485	0.81	0.013974431	11.3044535	0.044844427	0.006201792	
			90	4.61	0.06966876	0.282360729	0.06210053	0.006176609	
			145	4.80175	0.064626136	0.33571571	0.073036145	0.006122553	
			170	4.2488	0.064314895	0.39511471	0.071424362	0.006118926	
			195	4.09915	0.062115763	0.438902596	0.07491139	0.006107459	
			215	3.79375	0.058007243	0.477341145	0.073542198	0.006109897	
			235	3.652375	0.054869924	0.515919355	0.073492172	0.006108911	
			260	3.5835	0.051996801	0.76764229	0.074324862	0.006109139	
			275	3.57975	0.050426069	0.844171531	0.074242430	0.006094791	
			285	2.902375	0.053172951	0.648051835	0.074742827	0.006109755	
			315	3.272375	0.049666898	0.726528355	0.078005671	0.006099047	
			343	3.042159	0.046172765	0.81719481	0.0770574	0.006097304	
			375	2.761275	0.04019112	0.931485053	0.073802996	0.006098091	
			403	2.551319	0.037184997	1.01729905	0.072477477	0.006113186	
			415	2.383175	0.03620413	1.105745226	0.071505705	0.006115555	
			422	2.140544	0.033158468	1.18270824	0.069321726	0.006111676	
			240	2.084175	0.031642339	1.245210497	0.074749612	0.006115136	
			90	5.4086	0.080897372	0.351022274	0.076289668	0.006109222	
			145	5.1457	0.073890531	0.47408012	0.075365799	0.006103778	
			170	5.01002	0.070667964	0.483480201	0.080491361	0.006101401	
			215	4.7411	0.071208643	0.578718054	0.080249361	0.006098981	
			240	4.6111	0.06999577	0.530970469	0.08072362	0.006094124	
			260	4.4536	0.067382608	0.692625827	0.079956019	0.006090382	
			275	4.3319	0.065757077	0.735720259	0.080134579	0.006092563	
			285	4.2841	0.064627868	0.767545573	0.079663183	0.006091421	
			315	4.0271	0.061121853	0.863375722	0.084215771	0.006076669	
			325	3.9469	0.059964591	0.89601574	0.084245606	0.006076766	
			343	3.78958	0.057645215	0.964660489	0.085160463	0.006067345	
			360	3.6536	0.055452996	1.033648086	0.087134712	0.006060131	
			375	3.5219	0.053454906	1.082541789	0.086509363	0.006052404	
			415	3.1531	0.047957596	4.41190607	0.082506289	0.00604561	
			422	3.08308	0.046887058	5.2294978	0.081855827	0.006110489	
			445	2.52	0.038247628	9.09119004	0.073896484	0.006138788	
			90	1.847	0.029550846	1.10554371	0.06413586	0.006097726	
			145	1.6945	0.026420516	0.25111161	0.071740497	0.006091407	
			170	1.548675	0.02324417	0.340489598	0.077962526	0.006099997	
			195	1.3733	0.021525089	0.335142557	0.080513263	0.006094009	
			215	1.24675	0.019653294	0.3696469	0.086423894	0.006074261	
			235	1.138875	0.017799245	0.29941077	0.089014529	0.006071261	
			240	1.023875	0.016250518	0.411862061	0.08229946	0.006071171	
			260	0.87	0.01591596	0.476065755	0.084760488	0.006074926	
			275	0.72225	0.01471028	0.504230529	0.084999283	0.00607115	
			285	0.59875	0.011480854	0.524820355	0.084409071	0.00607079	
			315	0.491975	0.008178018	0.89899076	0.080959822	0.006059248	
			325	0.412375	0.0067862767	0.614312376	0.080487112	0.006052147	
			343	0.278459	0.004976089	0.658013994	0.090140979	0.006057045	
			360	0.412	0.006797147	0.701170977	0.092153364	0.006049575	
			375	0.012725	0.000972115	0.82111427	0.091179113	0.006052517	
			403	0.377619	0.05724594	1.44926701	0.088985229	0.006057863	
			415	0.368875	0.056825148	1.78847583	0.088163011	0.006056206	
			422	0.360244	0.056464173	2.017317307	0.088762105	0.006056149	
			445	2.8905	0.04531221	3.02951218	0.07293254	0.006052193	
			460	2.420	0.0313464675	3.91777828	0.066487134	0.006112132	
			475	1.5405	0.023818114	5.12723732	0.058144131	0.006116374	
			485	0.876	0.013235658	6.378131595	0.056113892	0.006104466	
			505	0.2115	0.003210009	8.693418784	0.030520493	0.006232147	
			90	5.4444	0.063231051	0.203922274	0.074738864	0.006108088	
			145	5.2002	0.078932713	0.344848682	0.076380233	0.006105131	
			170	5.0812	0.077126575	0.387801734	0.077440345	0.006099173	
			195	4.9572	0.075339548	0.495951146	0.084754354	0.006091443	
			215	4.8544	0.073870289	0.478523898	0.084615352	0.0060858	
			235	4.7484	0.07200496	0.52088294	0.081836831	0.006084551	
			240	4.6114	0.069999125	0.59122338	0.081231097	0.006084972	
			260	4.5288	0.068270607	0.684415478	0.081424553	0.006083861	
			285	4.4694	0.067834950	0.669494823	0.081766605	0.006084651	
			315	4.2924	0.065148465	0.765633859	0.080707312	0.006072156	
			325	4.2118	0.064424096	0.80487208	0.080621028	0.006069477	
			343	4.10704	0.062547532	0.873321787	0.080467006	0.006062614	
			360	4.0134	0.060912092	0.945984525	0.08080846	0.006054488	
			375						

	505	0.277	0.004204204	9.143710609	0.031484898	0.006248061
	525			13.02754811	0.021979	0.006208125
	50	5.4306	0.08423839	0.363448805	0.07606072	0.006348311
	145	5.13855	0.07299102	0.431390259	0.076191349	0.0061031
	170	4.9938	0.07572905	0.460488063	0.07654115	0.006292004
	195	4.84155	0.074883256	0.508588181	0.08113739	0.006086277
	215	4.71435	0.07152662	0.544203994	0.07997238	0.006090801
	235	4.58215	0.06949214	0.578031524	0.08007231	0.006090451
	260	4.43106	0.06654246	0.626180646	0.078675911	0.006091729
	275	4.31039	0.06312132	0.64669701	0.076785161	0.006091151
	285	4.23135	0.064221866	0.678313201	0.078415053	0.006092642
	315	4.02635	0.06050699	0.746601419	0.080424081	0.006073321
	325	3.2895	0.059631211	0.770961636	0.083427328	0.006029442
	343	3.786606	0.052471206	0.816672073	0.085042491	0.006073992
	360	3.4486	0.055177192	0.862330505	0.087081407	0.006067221
	375	3.53395	0.053466311	0.879607907	0.086529865	0.006068025
	415	3.17835	0.048239821	0.856650699	0.082483148	0.006068134
	422	3.11886	0.047203108	0.844828179	0.08272579	0.006063324
	443	2.587	0.05926453	8.337797663	0.075091949	0.006106931
	460	2.017	0.05061328	12.23847506	0.066536308	0.006135109
Fourier and Caollax Number analysis for the polycarbonate data from the second set of experiments	475	1.447	0.011942031	18.03273073	0.056728409	0.006117258
	490	0.877	0.011310782	26.50409137	0.045337146	0.006204385
	505	0.937	0.004605513	80.97027292	0.034245682	0.006246101
	Thermal diffusivity (m^2/s)					
high delay time setting	Rowlength (mm)	Representative shear rate (1/sec)	Temperature (K)	Zero shear viscosity (Pa-sec)	Viscosity (Pa-sec)	
	30	141.5780455	518.4536291	233.0315609	224.1389427	
	145	184.6474712	619.8142299	231.2733148	220.0794617	
	170	208.2696806	620.2808989	226.6580147	216.1605121	
	195	235.8150007	620.5233938	227.4388014	213.807528	
	215	259.2813728	620.5408584	227.4350745	212.5072374	
	235	285.412106	620.6913715	226.3021361	210.638611	
	260	321.6145699	620.8207388	225.8522661	208.1935138	
	275	351.6148842	620.8651151	225.9333232	207.1266101	
	285	362.4055447	620.793349	225.9872456	206.3420078	
	325	439.8872112	620.4639144	226.0795732	204.4175098	
	343	473.7895642	619.9683299	230.432617	204.4481664	
	360	521.1183866	619.7160282	231.8101223	203.264375	
	375	617.5220317	619.7281665	231.538678	198.110866	
	403	1185.408008	620	230.361279	171.485034	
	415	1736.828522	620	230.2617279	160.1008102	
	422	2070.802041	620	230.2617279	151.2667861	
	445	3668.242869	620	230.2617279	121.1826174	
	460	5313.105139	620	230.2617279	105.7288055	
	475	7489.447006	620	230.2617279	81.2606909	
Base setting	490	11112.39849	620	230.2617279	63.65084795	
	505	16508.13251	620.7863298	226.6355981	48.4088624	
	531	26456.49315	621.88133331	219.1956484	31.20505291	
	140	160.8093576	621.4028931	222.782867	214.609499	
	145	177.5170208	621.4826535	222.365978	212.1491381	
	170	197.303821	621.8344508	220.5382113	209.6789305	
	195	219.7238224	621.6287096	221.6153795	209.2240704	
	215	218.889318	621.9281332	220.9539045	207.1827238	
	235	259.7506227	622.0515459	219.6757463	205.8368177	
	260	289.4313486	622.1630961	218.8457446	203.7488514	
	275	307.2174695	622.1757577	218.7084616	202.2166103	
	285	320.3083905	622.2071051	218.6202569	202.0741626	
	315	364.2193733	621.9187305	220.074568	201.321352	
	325	373.6379742	622.124213	219.0453019	199.7451465	
	343	409.8614567	621.7951787	220.7412175	199.8046533	
	360	460.660933	621.523319	222.1338074	199.9050575	
	375	562.7564417	621.5314315	222.1116	195.945194	
	403	1021.291061	621	224.9020212	178.973527	
	415	1299.125416	621	224.9020212	169.747979	
	422	1498.830713	621	224.9020212	164.068468	
	445	2365.422958	621	224.9020212	142.8341707	
	460	3182.450685	621	224.9020212	127.245886	
	475	4178.877031	621	224.9020212	113.3074315	
	490	5748.441222	621	224.9020212	95.45732445	
	505	144.6573778	624.2614468	208.1633229	201.0512643	
	145	127.4750951	624.3090544	208.120204	199.1013613	
	170	202.7749637	624.6793282	206.3676051	196.1631166	
	195	225.9297781	624.2360273	206.1060133	197.479461	
	215	245.3348399	624.7919384	206.000621	194.1794461	
	235	266.867384	624.8134507	205.7257958	193.2046627	
	260	298.4013634	624.9452275	205.079093	191.1484998	
	275	315.489014	624.6940999	205.3460631	190.833143	
	285	329.1829152	624.9046207	205.2904703	190.1800212	
	315	374.0418555	624.2984556	208.033565	190.894211	
	343	428.8403174	624.6912354	209.20539	189.7931179	
	375	481.3162218	621.8276093	210.5229714	188.4949755	
	403	540.2217262	624	209.654622	185.5216393	
	415	567.7407381	624	209.654622	184.4811776	
	422	584.3783888	624	209.654622	183.838506	
	445	647.2862988	624	209.654622	181.729986	
High piston speed setting	460	180.4118074	621.4750995	222.8139132	212.1271101	
	145	225.040311	621.5807584	221.8546465	209.4706973	
	170	246.7527782	621.9170615	220.088049	208.339996	
	195	276.4628113	621.2030462	219.5747138	201.872136	
	215	313.2749006	622.0997485	219.1707821	202.408868	
	235	357.1581889	622.2533635	218.7322999	200.1876501	
	260	373.3205646	622.2618507	218.4401368	199.120662	
	275	394.0990227	622.2892983	216.1998326	198.1760108	
	315	446.1325589	621.9851695	219.7333733	197.4222322	
	325	464.1343232	622.1843879	218.7265094	195.8112237	
	343	489.2574557	621.8414412	220.510355	195.7280294	
	360	524.9659596	621.5564519	222.4802289	195.280212	
	375	670.5015328	621.5533894	222.0022827	192.305445	
	415	7277.616662	622	219.6837059	142.308019	
	422	8695.102722	622	219.6837059	134.038019	
	445	4671.882138	622	219.6837059	105.324792	
	460	6675.209513	622	219.6837059	86.7994884	
	475	126.183388	620.7254099	226.194831	215.917826	
	145	156.7082338	620.757894	226.1865061	216.9631041	
	170	172.8277441	621.0916873	224.417744	214.7434316	
Low piston speed setting	195	195.1433176	620.9815489	224.499984	214.2380955	
	215	206.4830863	621.1543007	224.087712	212.4163021	
	235	223.2861809	621.2206178	223.787588	211.4346204	
	260	246.0862574	621.3451215	223.089728	209.2653369	
	275	260.9266261	621.3369482	223.1281339	208.8145619	
	285	272.2144493	621.3463955	223.0797229	208.2652955	
	315	305.7607076	621.0205224	224.7932215	208.6626153	
	325	331.7163256	621.1511088	224.1045229	206.804787	
	343	344.1276607	620.7924553	226.026382	207.320666	
	360	384.955015	620.5063135	227.330087	207.902051	
375	457.805462	620.457154	227.637966	203.232884		
	Flowlength (mm)	Representative Shear Rate (1/Pa-sec)	Temperature (K)	Zero shear viscosity (Pa-sec)	Viscosity (Pa-sec)	
	403	748.472136	621	224.9020212	189.9150356	
	415	924.096484	621	224.9020212	182.4099671	
	422	1044.981235	621	224.9020212	178.1145939	
	445	1556.398135	621	224.9020212	162.728862	
	460	2018.544295	621	224.9020212	150.4348872	
	475	2512.270442	621	224.9020212	137.784329	
	490	3178.802513	621	224.9020212	124.467306	
	505	4165.914212	620.6697842	226.6469124	110.614082	
	525	136.7010609	621.3781395	222.9232724	215.0089757	
Low short size setting	531	177.2243275	621.4315181	222.6226215	212.6025365	
	170	359.321852	621.7487668	220.9810051	209.9749033	
	195	224.8957139	621.5078415	222.245378	209.8176682	
	215	245.8983706	621.2124589	221.0136215	208.4270232	
	235	271.2861056	621.7606151	220.9202431	206.379006	
	260	305.0891307	621.8371939	220.5238182	204.4182602	
	275	322.4477973	621.7970599	220.1214748	203.5448873	
	285	341.787173	621.787173	220.7826092	202.846967	
	315	396.2401541	621.3909098	222.859782	201.1545179	
	325	415.1306003	621.5313771	222.957449	200.293219	
	343	452.3381461	621.104782	224.3487821	200.843967	
	360	490.6183662	620.753442	226.102091	200.2435009	
	375	672.4643144	620.678788	226.6187079	199.178213	
	403	1712.034378	621	224.9020212	158.1016133	
	415	2280.460894	621	224.9020212	144.3488424	
	422	2706.416132	621	224.9020212	131.78906	
	445	4589.329523	621	224.9020212	106.3551199	
	460	6636.97386	621	224.9020212	87.49812225	
	475	9313.90996	621	224.9020212	67.72723858	
	505	19387.939316	620.62392	226.3008842	45.95363402	
High short size setting	525	128.54510245	620.216977	226.6236557	218.3604812	
	145	162.1702333	620.7344855	226.3114073	216.7817343	
	170	180.201031	621.0551178	224.5579989	214.2128719	
	195	200.8050608	620.9579685	225.1242431	213.64551369	
	215	218.2284689	621.1213395	224.9251577	211.959937	
	235	237.3897081	621.1883972	223.9082728	210.67878	
	260	263.6065671	621.2908208	223.3486903	208.803773	
	275	280.7869353	621.2995121	223.263394	208.013341	
	285	298.7368496	621.3063966	223.2881373	207.396809	
	315	331.2914733	620.9731323	223.0420402	206.9382844	
343	374.646939	620.7348244	226.309336	206.078066		
360	402.8879684					

Low gas pressure setting	403	482.0124549	630	230.2617279	204.2104426
	422	521.728951	630	230.2617279	202.2962639
	443	106.9807157	620.6934458	226.5298382	209.9635924
	145	119.7736234	620.6445152	226.7908995	218.4531044
	170	158.0721439	620.9550505	225.1369651	215.222044
	195	179.3893539	620.8604917	225.4411810	215.448608
	215	187.6022155	620.9528594	224.9397861	213.6460889
	235	212.9920996	621.0517796	224.4022621	212.2897991
	260	265.3564248	621.1819015	224.4188905	210.1544571
	275	265.1905846	621.1488547	224.1163955	209.4698922
	285	278.4109539	621.1495112	224.1279575	208.8122151
	315	322.402494	620.7596157	226.0177125	208.5473204
	325	339.4824148	620.8965831	225.4496786	206.9869094
	343	371.1068354	620.5135643	227.4908269	207.1807918
	360	403.8410824	620.2252306	229.0420038	206.895832
	375	475.7954847	620.2038848	229.1583645	203.6230072
	403	780.1453236	620	230.2617279	192.4376574
	415	964.4190406	620	230.2617279	184.3963445
	445	1631.853532	620	230.2617279	162.994008
	460	2116.482004	620	230.2617279	150.4876124
	490	3551.842994	620	230.2617279	122.9813642
	505	4595.224498	620.2708322	228.7964214	108.1620707
High gas pressure setting	525		621.1983339	219.7959418	88.9589472
	90	186.8254632	621.4110317	222.008763	212.1835068
	145	222.9687795	621.46095	221.7517207	209.4824801
	170	261.5944835	621.9862092	219.7759405	206.8022505
	195	262.3881187	621.7736957	220.8524711	206.7443216
	215	279.5222191	622.1027526	219.1553556	204.382729
	235	298.0126771	622.1756573	218.776234	203.2405991
	260	322.7639662	622.3330017	217.976661	201.4103111
	275	338.6693784	622.3357239	217.862585	200.6755461
	285	349.565449	622.3600438	217.839703	200.1776546
	315	385.7884557	622.0137904	219.5097971	199.8591549
	325	398.164557	622.230996	218.497958	198.707784
	343	422.1857608	621.8711556	220.148061	198.3561542
	360	446.2863448	621.5687321	221.9171263	199.103925
	375	713.7799297	521.5473046	222.0287054	188.1991543
	415	1991.326792	622	219.8837059	148.6395965
	422	2383.06236	622	219.8837059	140.103581
	445	4287.044539	622	219.8837059	109.8652124
	460	6274.501859	622	219.8837059	89.5760021
	475	9177.817275	622	219.8837059	71.32349266
	490	13413.53303	622	219.8837059	54.95722591
	505	19590.48744	622	219.8837059	41.32246193
Fourier and Capillary number analysis for the polycarbonate data.					
		Thermal Diffusivity (m ² /s)			
		n			
		Ts(K)			
		T(Pa)			
		R (Pa-m)			
		R (m)			
High delay time setting	Rowlength (mm)	Surface Tension (N/m)	Ca*tau	Unmodified Fractional Coverage	Adjusted Fractional Coverage
	90	1	1.61.38529	0.697234689	0.612.905664
	145	1	178.48384	0.724263224	0.636.129818
	170	1	187.0013187	0.721313187	0.634.438527
	195	1	196.96688	0.731492284	0.639.207824
	215	1	198.09517	0.721607207	0.629.174291
	235	1	115.644641	0.724449991	0.630.122225
	260	1	128.9432	0.727618456	0.634.400592
	275	1	137.8483	0.71737548	0.624.713564
	285	1	144.49001	0.724800135	0.634.633995
	325	1	172.9615	0.72024622	0.626.897292
	343	1	188.5993	0.726672795	0.628.568122
	360	1	203.8106	0.728904766	0.630.964354
	375	1	242.871	0.720264006	0.622.224677
	403	1	428.35785	0.722673635	0.626.599124
	415	1	534.8777	0.718447447	0.623.043945
	422	1	603.7039	0.718957268	0.623.865945
	445	1	865.501887	0.71018897	0.623.049516
	460	1	1041.203	0.66145794	0.586.885125
	475	1	1223.11	0.64457964	0.586.610702
Base setting	490	1	1496.386	0.647392231	0.460.90148
	505	1	1550.11	0.624885193	0.594.750209
	525	1	1718.194	0.590755298	0.574.876298
	90	1	18.80377	0.67779472	0.596.985116
	145	1	73.28249	0.703137029	0.626534552
	170	1	80.3721	0.691550379	0.614.425569
	195	1	89.20291	0.699264593	0.614.338139
	215	1	95.93832	0.701241289	0.620579037
	235	1	103.6669	0.702084104	0.621.314768
	260	1	113.92748	0.702724639	0.623.427186
	275	1	120.8309	0.695427813	0.614.964781
	285	1	125.5526	0.700071162	0.620.126977
	315	1	141.8815	0.69118994	0.606.69515
	325	1	146.8089	0.697862335	0.614.302011
	343	1	158.3852	0.699151367	0.611.566536
	360	1	169.9351	0.691680165	0.608.400796
	375	1	217.9243	0.697149418	0.610.545835
	403	1	353.7884	0.689726276	0.605.956019
	415	1	427.5113	0.697779110	0.613.366148
	422	1	475.2794	0.691462878	0.609.743872
Low delay time setting	445	1	656.0722	0.652087131	0.576.552503
	460	1	701.1918	0.645144384	0.576.951533
	475	1	935.6253	0.62878281	0.572.406124
	490	1	1084.743	0.619778545	0.573.334121
	505	1	56.77421	0.65407118	0.598.461527
	145	1	70.83516	0.665678326	0.599.140182
	170	1	77.6793	0.66584514	0.594.440501
	195	1	86.67424	0.666937007	0.588.985106
	215	1	92.7868	0.665206569	0.599.952632
	235	1	100.3132	0.66760907	0.593.108795
	260	1	110.3861	0.664453162	0.590.003029
	275	1	117.205	0.659576701	0.589.032839
	285	1	121.851	0.651268629	0.579.533241
	315	1	138.6866	0.648118179	0.579.053152
	343	1	155.1274	0.650552792	0.577.947121
	375	1	176.7894	0.651622842	0.573.855301
	403	1	195.0574	0.648421831	0.574.420385
	415	1	203.9897	0.648282304	0.575.804311
	422	1	209.8574	0.631809994	0.561.203589
	445	1	227.9113	0.628464875	0.561.110918
High piston speed setting	460	1	74.49296	0.696792714	0.622.994394
	145	1	93.6285	0.70869577	0.626.606809
	170	1	99.89899	0.707882141	0.623.116543
	215	1	117.9887	0.708953373	0.628.606912
	235	1	126.8683	0.705231156	0.625.150994
	260	1	138.7052	0.699458032	0.619.01413
	275	1	146.3349	0.70143672	0.621.502092
	285	1	151.9856	0.704677476	0.624.914391
	315	1	170.449	0.699915707	0.615.675986
	325	1	175.9381	0.702106235	0.618.660579
	343	1	189.0139	0.702480258	0.617.278395
	360	1	201.9544	0.69891505	0.605.680138
	375	1	206.7791	0.70152703	0.613.020646
	415	1	617.8496	0.69931134	0.594.424844
	422	1	699.8563	0.617488469	0.555.612842
	445	1	957.5076	0.621183955	0.54909747
	460	1	1132.757	0.613461089	0.54694717
	90	1	53.67508	0.68239709	0.605.132999
	145	1	65.95865	0.69807312	0.618310595
	170	1	71.87948	0.702729613	0.626.979951
Low piston speed setting	195	1	79.0798	0.706231596	0.619807702
	215	1	84.84892	0.707991626	0.623030197
	235	1	91.22182	0.70932366	0.62409676
	260	1	99.74841	0.708581543	0.623820595
	275	1	105.3984	0.701737398	0.618774115
	285	1	109.3019	0.702160712	0.621750842
	315	1	122.7489	0.690731214	0.6013721092
	325	1	126.9115	0.702222289	0.614734976
	343	1	136.43508	0.702054796	0.613431817
	360	1	145.908	0.702598966	0.610245432
	375	1	179.2741	0.702197434	0.610018318
High delay time setting	Rowlength (mm)	Surface Tension (N/m)	Ca*tau	Unmodified Fractional Coverage	Fractional Coverage
	403	1	272.7795	0.692279374	0.602.384145
	415	1	325.2124	0.697874714	0.608.713703
	422	1	358.3128	0.697550677	0.608.808576
	445	1	490.7902	0.660354528	0.589.029428
	460	1	592.31	0.656144062	0.586.994328
	475	1	705.3968	0.64846623	0.582.502502
	490	1	824.1952	0.644127911	0.588.809149
	505	1	961.9564	0.63314738	0.602.794334
Low shot size setting	90	1	57.1792	0.707807778	0.599.842514
	145	1	78.21091	0.701208446	0.624.427094
	170	1	81.32367	0.702841462	0.625.401118
	195	1	93.19472	0.701430598	0.618.735064
	215	1	99.30109	0.707064774	0.626.481722
	235	1	108.4877	0.704267557	0.622.408924
	260	1	120.8541	0.704437099	0.622.709602
	275	1	129.1346	0.697208965	0.615.066433
	285	1	134.8938	0.700352376	0.618565771

	315	1	154.7778	0.689056354	0.601985642
	325	1	161.1072	0.695702575	0.612380377
	343	1	175.4038	0.701786078	0.613393072
	360	1	189.7778	0.698799327	0.607818482
	375	1	211.1907	0.695270999	0.608240511
	403	1	232.3336	0.686868461	0.597302016
	415	1	236.8535	0.693377846	0.606404724
	422	1	239.1277	0.692602561	0.601882791
	445	1	266.8936	0.621449629	0.547718591
	460	1	1141.872	0.627848174	0.551193891
	490	1	1471.544	0.61580335	0.57387239
	505	1	1519.459	0.616109316	0.585486794
High shot size setting	90	1	94.81884	0.672382415	0.595057721
	145	1	66.15975	0.699646312	0.616575586
	170	1	74.84828	0.705181268	0.620507165
	195	1	82.81459	0.701738315	0.615087834
	215	1	88.48699	0.702322545	0.620018739
	235	1	96.72071	0.705898509	0.620493216
	260	1	106.4813	0.701207358	0.616159921
	275	1	113.16638	0.708678887	0.613374344
	285	1	117.463	0.703816372	0.619104597
	315	1	132.9532	0.698529973	0.608637112
	343	1	148.8746	0.708323551	0.613653145
	360	1	159.86277	0.704325992	0.611624542
	375	1	169.29777	0.708604945	0.610304845
	403	1	189.6538	0.685132306	0.595171205
	422	1	203.7982	0.673952105	0.584690204
Low gas pressure setting	90	1	45.55964	0.70186614	0.624460433
	145	1	59.21402	0.708966546	0.624573636
	170	1	66.15588	0.715469093	0.634104364
	195	1	74.54496	0.709629477	0.621947905
	215	1	83.60719	0.715164545	0.625296795
	235	1	89.43828	0.705424738	0.618689426
	260	1	100.1679	0.704533297	0.617989777
	275	1	107.3588	0.710230389	0.621600367
	285	1	112.3964	0.708774738	0.622705517
	315	1	129.8426	0.703280722	0.612148342
	325	1	135.538	0.714882142	0.621214526
	343	1	148.1742	0.712119017	0.620278608
	360	1	160.8242	0.714311194	0.620232198
	375	1	184.4823	0.710045123	0.613971599
	403	1	287.7893	0.709871812	0.613912036
High gas pressure setting	415	1	342.8023	0.70128536	0.611508215
	445	1	515.1317	0.699779951	0.612790310
	460	1	620.4521	0.707338813	0.635223869
	490	1	862.1688	0.662368545	0.615250058
	505	1	990.8788	0.644371148	0.609271275
	525	1	1547.152	0.637470269	0.615593269
	90	1	77.11789	0.682723977	0.608117265
	145	1	96.78767	0.704218021	0.620020571
	170	1	97.09124	0.699624337	0.621210223
	195	1	105.1477	0.701748647	0.620411257
	215	1	119.8468	0.70438037	0.621480779
	235	1	117.48077	0.70243286	0.622355627
	260	1	126.1192	0.704390352	0.624614442
	275	1	131.8378	0.697702091	0.617437129
	285	1	135.7193	0.698056443	0.618641189
	315	1	149.2311	0.696050432	0.611981551
	325	1	153.0534	0.694936722	0.611489194
	343	1	162.4658	0.698370737	0.611328246
	360	1	171.7335	0.698107561	0.611026555
	375	1	209.641	0.69501785	0.608487955
	415	1	573.3288	0.688227046	0.599391857
	422	1	646.8597	0.681293654	0.599306357
	445	1	956.0339	0.641569364	0.566497315
	460	1	1102.71	0.613927755	0.547390775
	475	1	1285.693	0.610523835	0.55579525
	490	1	1456.553	0.610874927	0.565537785
	505	1	1610.339	0.60715379	0.574697308

18 Appendix K: Data for PS

Fourier and Capillary number analysis using polystyrene data from the second set of experiments										
Thermal Diffusivity (m ² /s)										
n										
10 (s)										
1730										
8 (Pa-sec)										
1.68E-06										
0.00033										
0.000000075										
7.5931										
Rowlength (mm)										
Residual Time (sec)										
Fourier #										
Ln (m/sec)										
CMOLD character										
Tmelt (deg K)										
Solid Layer Fractional Coverage										
Adjusted Radius (mm)										
high piston speed setting	90	4.9947	0.009290161	0.14380893	531.88667	0.04196873	0.0021321			
	145	5.0392	0.00383773	0.28828238	531.244708	0.04620891	0.00262891			
	170	5.0307	0.00319921	0.34808138	531.2160146	0.04650592	0.00262036			
	195	4.9632	0.0023157	0.51955773	531.4157354	0.04619302	0.0026016			
	215	4.9072	0.00912741	0.662061885	531.4785738	0.04628032	0.00260139			
	260	4.7227	0.00878424	1.141218272	531.5138613	0.04601391	0.002619241			
	275	4.6432	0.008138363	1.368474749	531.4588205	0.04726024	0.00218113			
	285	4.5852	0.008132489	1.54310599	531.3514606	0.04713564	0.00219727			
	315	3.816	0.007201774	2.20214173	531.4054448	0.0444139	0.00270137			
	325	3.542	0.00588113	2.505789253	531.4834954	0.0429436	0.00212158			
	343	3.048	0.00552772	3.11543938	531.461189	0.0426279	0.00220122			
	360	2.581	0.00405413	3.822086297	531.518905	0.0383161	0.00227208			
	375	2.172	0.004019928	4.588712104	531.14346	0.0318864	0.00230084			
	460	0	0	12.8307516	529.7277778	0.00063	0.00230084			
475	0	0	15.3824268	529.7277778	0.00063	0.00230084				
490	0	0	18.4507267	529.7277778	0.00063	0.00230084				
low piston speed setting	90	5.7238	0.010644289	0.00942323	531.017638	0.04627042	0.00210392			
	145	6.0428	0.01123963	0.145111314	530.734156	0.05124396	0.00218561			
	170	6.1078	0.01186031	0.317289789	530.724687	0.0522088	0.00218602			
	195	6.1228	0.01138941	0.232073459	530.8096124	0.05268578	0.00218953			
	215	6.0988	0.01134781	0.264893402	530.842021	0.05295328	0.00219787			
	235	6.0438	0.01222863	0.314662955	530.8851129	0.05264296	0.00217223			
	260	5.9278	0.01105273	0.390138236	530.8880342	0.051378846	0.002178846			
	285	5.8348	0.01085275	0.443856419	530.8349887	0.05405436	0.00217598			
	295	5.7628	0.010718629	0.481771534	530.7882838	0.05414736	0.00217361			
	315	5.4988	0.010227788	0.626094381	530.7435146	0.05149304	0.00217541			
	325	5.3948	0.010334144	0.682111515	530.703888	0.05138894	0.00217181			
	343	5.18744	0.009448018	0.795559141	530.4446101	0.05110509	0.002172891			
	360	4.9678	0.009240126	0.921939346	530.429973	0.05126018	0.002178249			
	375	4.7548	0.008842964	1.048920452	529.994114	0.05181776	0.00218308			
Base Setting	415	2.86688073	0.005131864	1.479561633	529.7277778	0.005485263	0.0021326			
	435	1.052432176	0.001957265	1.91054056	529.7277778	0.021424007	0.00228161			
	460	0.617706487	0.001186136	2.178738087	529.7277778	0.017188614	0.00234494			
	475	0.38601841	0.000718709	2.478228804	529.7277778	0.015037006	0.00230706			
	490	0.21410377	0.000495483	2.8002551	529.7277778	0.01510601	0.00230706			
	500	5.2493	0.00976178	0.101999478	531.258522	0.04348087	0.002196878			
	145	5.4155	0.01006063	0.188027055	531.0533792	0.047454835	0.002196878			
	170	5.4532	0.010050928	0.234364936	531.102874	0.049481406	0.00219346			
	195	5.40155	0.010046003	0.303954936	531.1866098	0.0486198615	0.002193615			
	215	5.35555	0.009961343	0.3242332	531.2309652	0.04879706	0.002193151			
	235	5.3855	0.00981143	0.460760993	531.282077	0.049132153	0.002193404			
	260	5.1843	0.00905617	0.597574794	531.2965831	0.04913051	0.00219105			
	275	5.07355	0.009438622	0.698461077	531.214892	0.049719235	0.002190129			
	285	5.0055	0.009113942	0.775017228	531.1761573	0.049731647	0.002191471			
	325	4.67351	0.00809282	1.17483845	531.1271494	0.04950609	0.002190021			
	343	4.49281	0.008234928	1.416881431	530.9345487	0.04902097	0.002191446			
	360	4.3043	0.008006014	1.606687724	530.7385005	0.0485621	0.002192339			
	375	4.12155	0.007868184	1.872697264	530.7259203	0.047796051	0.002193887			
	445	0.674838901	0.002138455	4.092386935	529.7277778	0.021424007	0.00234494			
	460	0.602157876	0.001478789	4.782888862	529.7277778	0.017178015	0.002393894			
	475	0.240279379	0.000440554	5.590898982	529.7277778	0.01507606	0.00230507			
	490	0.145184448	0.000264789	6.514681204	531.0510891	0.01510891	0.00230779			
	500	7.3771	0.01371433	0.008939402	530.8784833	0.002169557	0.00230779			
	510	7.5668	0.010274269	0.1055452	529.90444	0.002393894	0.00230779			
	520	7.5931	0.01423194	0.262567334	529.994607	0.00252136	0.002154846			
	530	7.5185	0.014102269	0.356265411	530.049697	0.00289836	0.002153742			
	540	7.4465	0.014031309	0.454646703	530.069069	0.00305928	0.00215929			
	550	7.29385	0.01316658	0.934503938	530.020401	0.02137495	0.002148771			
575	7.2365	0.013445408	1.097761131	530.0163386	0.02161845	0.00215981				
585	7.00285	0.013030267	1.539953594	529.9716036	0.02193393	0.002148038				
595	6.91881	0.012786582	1.732738895	529.9051895	0.02205185	0.00215231				
610	6.74713	0.012449687	2.167018545	529.8883167	0.02220807	0.00214899				
high delay time setting	90	5.1671	0.01211483	2.6664142	529.6862985	0.02269311	0.002147389			
	170	5.3936	0.01181285	3.201923292	529.2647384	0.02425163	0.002159161			
	345	0.483196916	0.00000184	7.521399937	529.7277778	0.021424007	0.00218811			
	460	0.34867404	0.00014084	9.01850143	529.7277778	0.017188015	0.002393894			
	490	0.11197976	0.000112003	10.45551207	529.7277778	0.01507606	0.00230706			
	490	0.05513178	0.000102884	19.0243917	529.7277778	0.01503891	0.00230853			
	520	0.010317128	1.42411E-05	49.0604935	529.904935	0.01171449	0.00230779			
	530	3.8544	0.007187798	0.15143272	531.0554339	0.05152096	0.002159615			
	145	3.8864	0.007228718	0.227497996	531.0246475	0.05099528	0.00225097			
	170	3.8664	0.007212058	0.273272784	531.0688861	0.05099528	0.00225097			
	195	3.8014	0.00720618	0.329556274	532.0230882	0.05088784	0.00222715			
	215	3.7394	0.006955298	0.381893369	532.102425	0.05074064	0.00222544			
	235	3.6814	0.006810218	0.442810287	532.1336304	0.05060967	0.00222477			
	275	3.4574	0.059345215	0.520792054	532.0792054	0.04905718	0.00262006			
	285	3.3964	0.05817167	0.61037178	532.008039	0.04775948	0.002121424			
	325	3.1174	0.05178025	0.8811901876	530.1744188	0.04945139	0.002217249			
	343	2.96372	0.05051213	0.984704138	531.228952	0.03989348	0.002217249			
	360	2.8114	0.050210214	1.116203027	529.9907395	0.03973161	0.002159161			
	375	2.6674	0.049651374	1.24780518	530.923007	0.03981788	0.00215245			
	445	0.309	0.000514261	2.096401224	529.7277778	0.021424007	0.00231611			
	460	0	0	2.40502486	529.7277778	0.017188015	0.00234494			
	475	0	0	2.61531932	529.7277778	0.01507606	0.00230706			
	490	0	0	0.00070057	0.008468165	0.04145055	0.002104014			
	high gas pressure setting	90	5.2183	0.00970607	0.09846765	531.274514	0.0412877	0.002178015		
		145	5.3813	0.010167068	0.10861005	531.067662	0.04784607	0.002187402		
170		5.4183	0.010078608	0.227635408	531.1074559	0.04819461	0.002194611			
195		5.4283	0.01006668	0.347734245	531.176276	0.04879848	0.00220139			
215		5.4383	0.010078908	0.443104126	531.205441	0.04905485	0.00220884			
235		5.39323	0.010079698	0.562040688	531.226856	0.04936511	0.00221544			
260		5.2923	0.00984389	0.698473604	531.1189517	0.05066611	0.00222208			
285		5.2573	0.00977808	0.820470603	531.0472109	0.05136668	0.00222884			
315		5.1283	0.009513867	1.46793798	530.9332847	0.05193778	0.00223544			
343		4.9742	0.0092543	2.094002153	530.747438	0.05261478	0.00224204			
360		4.8171	0.009055136	2.518811908	531.4757051	0.05322053	0.00224877			
375		4.7623	0.008857896	3.015157899	529.999785	0.05319236	0.00225544			
445		0.800841874	0.001438369	4.983175795	529.7277778	0.021424007	0.00231611			
460		0.503530859	0.00080789	8.362773746	529.7277778	0.017178015	0.00234494			
475	0.319318014	0.000594343	10.40120579	529.7277778	0.01507606	0.00230706				
490	0.324590014	0.000337073	11.886696	529.7277778	0.01503891	0.00230853				
500	5.5839	0.01038676	0.13186636	531.090815	0.0454501	0.00215446				
510	5.7685	0.01017805	0.21928887	530.875879	0.049472409	0.002159615				
520	5.8301	0.010074809	0.277459333	530.8201383	0.05038894	0.00216611				
195	5.7676	0.01005286	0.339129861	530.820784	0.05060884	0.00216844				
215	5.7255	0.010036145	0.431728478	531.4148631	0.05062092	0.00217044				
235	5.6465	0.01003441	0.50709024	531.040284	0.05148205	0.00217255				
260	5.5499	0.01012285	0.649535573	531.084183	0.05163443	0.00217463				
285	5.4365	0.010162409	0.778790887	531.090484	0.05182895	0.00217684				
325	5.0785	0.00944608	1.172981247	530.924028	0.05185617	0.002178015				
343	4.9031	0.00920212	1.390730527	530.7821753	0.					

Low melt temperature	323	4.1948	0.007802344	1.017428742	547.453482	0.043893856	0.006209253	
	343	4.03712	0.007260258	1.196354006	547.299955	0.042940274	0.006212136	
	360	3.8793	0.007209912	1.394141207	546.938623	0.042179995	0.006210566	
	375	3.7248	0.006028142	1.595645894	546.504103	0.042218021	0.006214447	
	445	0.832773185	0.003132275	2.996001612	548	0.031424057	0.006281611	
	460	0.336849001	0.000779915	3.429034051	548	0.031771801	0.006291494	
	475	0.336937569	0.000626705	3.924655066	548	0.0315507866	0.006300507	
	490	0.2148617	0.000608095	4.491912707	548	0.03136951	0.006301651	
	505	0.137112995	0.000254313	5.141159126	548	0.031124097	0.006312097	
	90	5.3622	0.00973712	0.091323369	519.2441407	0.046311926	0.006202121	
High piston speed setting	145	5.6757	0.010563883	0.186658883	519.1262044	0.050592523	0.006186187	
	170	5.7382	0.010673073	0.256314667	518.9377195	0.05378125	0.006176885	
	195	5.7507	0.010696323	0.353861898	519.0029204	0.054248107	0.006175361	
	215	5.7247	0.010647968	0.454007191	519.0789901	0.054474616	0.006174619	
	235	5.6667	0.010540093	0.59282343	519.1320831	0.054508879	0.006175163	
	260	5.5492	0.01021533	0.81845425	519.1582138	0.054355468	0.006175001	
	275	5.4547	0.01043792	0.999378837	518.135743	0.054643278	0.00617439	
	285	5.3817	0.010009982	1.129925203	519.0804635	0.054858847	0.006173394	
	315	5.1147	0.009513361	1.662878693	519.0804395	0.055757346	0.006170431	
	325	5.0097	0.009318094	1.893978387	519.0299491	0.056138224	0.006171997	
Base setting	343	4.80054	0.008528142	2.387749804	518.971995	0.054860617	0.006171331	
	360	4.5792	0.008017329	2.872327317	518.7990236	0.054296651	0.006175202	
	375	4.3647	0.008118318	3.4077992195	518.34412	0.05414412	0.006177831	
	460	0.20394981	0.000379347	10.80111735	520	0.021424007	0.006281611	
	475	0.10077335	0.000374749	13.10706054	520	0.017718015	0.006293944	
	490	0.049720276	0.00016105	15.98281692	520	0.015507086	0.006300507	
	525	0.00961067	1.787286E-05	24.98189691	520	0.01510691	0.006301853	
	Low pressure setting	Rowlength (mm)		Representative shear rate (1/sec)	Zero shear Viscosity (Pa-sec)	Viscosity (Pa-sec)	Surface Tension (N/m)	Ca #
		30	71.20543396	3128.844326	447.7783881	0.01926192	3391.500801	
		145	143.65135	3348.042341	295.5004855	0.019276244	4354.65613	
		170	184.4556723	3338.43321	242.7222123	0.019269048	4839.88665	
		215	263.1833201	3335.648435	190.705568	0.019255847	5247.75045	
		235	335.2309728	3316.643243	169.0459603	0.019252647	5810.370176	
		260	578.0468467	3309.397662	117.0608912	0.019247167	6936.602236	
		275	693.217247	3319.248884	103.538919	0.01925444	7356.498129	
		285	752.4677887	3328.803314	95.47703708	0.019261487	7695.944065	
		315	1123.104625	3299.744014	74.29212167	0.019239466	8563.239478	
		325	1286.572797	3287.515944	68.182708	0.019238971	8887.545273	
		343	1372.767694	3284.444951	64.81318889	0.019232118	9132.14617184	
		360	1929.821243	3308.595347	51.17842152	0.019246599	10168.478721	
		375	2310.83396	3363.712748	45.30134724	0.01928466	10811.497727	
		445	6446.738534	3359.727283	22.83260607	0.019243902	12148.726132	
		475	7609.204217	3359.727283	20.12849999	0.019243902	16061.62396	
		490	8123.466418	3359.727283	17.2386999	0.019243902	18953.12689	
		505	45.76500653	3378.548515	597.7831693	0.019258939	2806.253314	
		145	73.66817648	3417.72817	452.5244905	0.019237759	3409.967006	
		170	91.39659796	3414.800225	386.4506459	0.019232549	3702.380125	
		195	113.3332208	3407.32827	345.8705702	0.019320145	4005.549056	
		215	134.6222276	3402.727889	309.8797076	0.019316875	4262.261664	
		235	159.9461295	3396.686897	272.2192717	0.019321917	4524.668897	
		260	198.3263045	3396.558379	241.060627	0.019312232	4882.534053	
		275	232.6666799	3403.815599	221.661287	0.019317094	5197.787902	
		285	245.565096	3410.263265	209.7421236	0.01932276	5242.860521	
		315	318.3423929	3416.434671	176.7123003	0.01925814	5743.901783	
		325	348.8739317	3413.130803	166.8291827	0.019324287	5903.467441	
		343	404.7303014	3430.150511	150.6327331	0.019333674	6230.929512	
360		468.4579311	3459.995818	136.914808	0.019335451	6553.138647		
375		532.472138	3521.542787	126.3323935	0.019403229	6874.089911		
445		733.648095	3519.727283	103.8703212	0.019423902	7824.843625		
475		857.281534	3559.727283	84.90132972	0.019423902	8441.039795		
490		1087.033325	3559.727283	77.8046433	0.019423902	8800.100515		
505		1239.31898	3559.727283	72.84688124	0.019423902	9148.449092		
525		1405.123119	3559.727283	65.18709912	0.019423902	9543.75188		
Low delay time setting	30	51.56635676	3446.749394	555.8314651	0.019278447	2941.499013		
	145	91.56068529	3375.828287	393.8732557	0.019295549	3695.140493		
	170	118.8001971	3367.10819	334.2535271	0.019290454	4006.952054		
	195	154.0971693	3355.893444	282.7367126	0.019282106	4461.612242		
	215	195.706651	3348.849328	246.8028515	0.019279615	4796.981899		
	235	233.6771209	3342.883155	215.2976349	0.019277383	5150.096688		
	260	302.0802441	3341.049872	181.237156	0.01927101	5622.548729		
	275	344.8008245	3349.527327	161.4509164	0.019270289	5926.741227		
	285	393.1762995	3357.308016	132.5483963	0.019283161	6177.858899		
	315	495.871717	3363.34951	115.733981	0.019288106	7030.951218		
	325	515.809051	3359.712084	101.4044363	0.019290289	7473.953083		
	343	557.2181751	3417.120562	90.40635002	0.019327132	7935.200664		
	375	700.170513	3443.74799	81.741458	0.019374085	8385.553451		
	445	2945.46021	3559.727283	50.70346469	0.019423902	10486.92324		
	460	2386.506524	3559.727283	45.1727041	0.019423902	12127.63487		
	475	2786.778452	3559.727283	40.6454542	0.019423902	13773.93739		
	490	3156.612389	3559.727283	36.38827144	0.019423902	15148.42902		
	505	3535.70297	3494.051063	573.8593599	0.019302882	2595.726835		
	145	88.795944	3525.399265	302.108523	0.019405524	3830.542286		
	170	113.3523216	3517.469394	254.7299133	0.019404279	4290.249524		
	195	181.7168915	3514.03384	258.2133622	0.019379796	4775.068911		
	215	232.068861	3510.891693	219.689692	0.019394672	5391.957639		
	235	296.214458	3502.617389	186.3146024	0.019389027	6113.151186		
	275	482.7199571	3510.62417	134.7924665	0.019394077	6615.369012		
	285	545.4472098	3515.543414	124.5214649	0.019398146	6881.539951		
	315	786.6443906	3524.724815	96.8440867	0.0194047	7948.89515		
	325	888.5229275	3520.480055	89.04414618	0.019401676	8042.616359		
	343	1108.143484	3538.470767	76.6959018	0.019411304	8618.478995		
	360	1381.891446	3565.744238	66.437117149	0.019424023	9242.363012		
	375	1634.840513	3627.286446	59.05305881	0.019476023	9816.419876		
	445	3739.737088	3559.727283	32.8389916	0.019423902	12638.12712		
	460	4506.220153	3559.727283	29.02343494	0.019423902	14068.52838		
	475	5405.66669	3559.727283	25.564665	0.019423902	14993.97666		
	490	6489.148781	3559.727283	22.40992151	0.019423902	15312.21966		
	505	9928.526538	3559.727283	17.6173118	0.019423902	17377.78655		
	High gas pressure setting	30	76.23940488	3253.611617	434.7439882	0.019204532	3417.855089	
145		114.2722125	3270.767646	338.2067346	0.019237128	3992.442527		
170		138.1182165	3259.99541	300.1594239	0.019205451	4265.5443		
195		186.1943453	3244.746073	266.088455	0.019197713	4549.807105		
215		232.7097826	3234.44891	241.473899	0.019188702	4787.434471		
235		273.4878086	3210.048218	219.0966139	0.019186282	5036.799729		
275		300.5411375	3217.403136	180.1332706	0.019191993	5573.719969		
285		321.672994	3217.403136	171.8420782	0.019199976	5715.17248		
315		435.091217	3259.592929	141.1356036	0.019208141	6313.312263		
343		496.9617676	3284.183915	129.5169991	0.019227795	6612.708313		
360		563.112794	3321.777924	119.1547217	0.019252776	6913.788601		
375		629.4003458	3391.787471	111.3601256	0.019308704	7214.012905		
445		1407.051455	3559.727283	79.8310601	0.019423902	8681.274848		
460		1167.746054	3559.727283	74.05768118	0.019423902	8887.886167		
475								

	215	173.3634721	3324.784812	261.1560179	0.0192548789	4637.535392
	235	207.408275	3311.783113	232.0673372	0.019248973	4933.716765
	275	297.3057842	3315.138259	189.0886229	0.01921313	5379.143985
	285	315.3153387	3322.124673	172.9672333	0.01921728	5731.250517
	375	466.1145347	3326.796679	135.6371795	0.019260306	6482.126127
	383	477.898827	3346.19324	121.8243932	0.019274863	6858.873024
	360	636.3393016	3379.392242	110.1681634	0.019299563	7214.354477
	375	730.1972043	3441.744817	101.1297412	0.019346714	7379.667233
	445	1354.880379	3559.727263	66.84301323	0.019423932	9404.60238
	475	1769.642922	3559.727263	55.57888874	0.019423932	10245.99854
	490	2025.091938	3559.727263	50.62500466	0.019423932	10683.47442
high melt temperature	90	61.37973146	1781.538652	385.4686335	0.019274438	3581.267893
	145	101.8209457	1788.019379	287.6259476	0.019274412	3906.598403
	170	127.5212468	1778.699703	250.9220013	0.0192874395	4384.762083
	195	159.7020555	1773.8919139	218.4242423	0.0192874394	4581.267893
	215	191.2173179	1770.045546	195.2034195	0.019274374	3831.24835
	235	228.9178384	1767.327356	174.1874678	0.019274383	4099.100991
	275	338.1726592	1768.705783	138.2380611	0.0192874389	4658.887177
	285	339.0836679	1769.705783	130.468928	0.01928134	4808.015139
	315	514.5108421	1773.108573	103.077342	0.017636448	5444.630101
	345	664.1097525	1775.399336	82.63761231	0.017640779	5753.171385
	360	704.6266604	1786.43126	63.9981428	0.01763636	6035.271897
	375	806.12873	1807.23183	50.8413995	0.017687154	7365.474652
	445	1497.616744	1841.283171	50.76084775	0.017735057	7285.746564
	475	1955.921124	1793.172906	45.2565941	0.017585633	8104.5417
	490	2238.168017	1793.572906	37.85871108	0.017585635	8826.716072
low melt temperature	505	2937.265219	1793.572906	34.53892801	0.017585635	9218.718889
	90	46.2434236	5493.302099	308.8238024	0.020487105	4429.633161
	145	94.2367948	5418.894281	463.2713533	0.020487999	4465.29058
	170	120.292869	5463.459509	376.3403568	0.020514012	4607.893018
	195	179.828979	5448.187127	309.8238024	0.020511451	5181.563166
	215	232.9170361	5530.37255	255.4457204	0.020505765	6074.064884
	235	301.4470246	5517.990746	214.564553	0.020498408	6801.712178
	260	416.186462	5511.903791	172.324816	0.020497469	7321.138967
	275	505.0812691	5517.138349	151.0373693	0.020490839	7787.734083
	285	574.742329	5530.028524	138.5302942	0.02050916	8115.343643
	315	846.7121186	5548.5121709	106.0862927	0.020511651	9164.933177
	345	961.0517147	5641.831613	97.03860404	0.020608711	9536.534849
	363	1145.93827	5555.112057	87.1246169	0.02051566	10251.23867
	380	1311.847781	5596.14654	75.51741311	0.020513013	10955.95795
	383	1833.830013	5704.901405	62.6421762	0.020527913	11378.67976
	460	3399.18406	5319.815466	28.67336075	0.020418035	16181.318

	475	0.515202825	0.4996951
	490	0.496022157	0.4809135
High shot size setting	90	0.479814409	0.6133332
	145	0.678385666	0.6297322
	170	0.678434498	0.6200527
	195	0.675156662	0.6233538
	215	0.675144517	0.6233224
	235	0.678017961	0.6276351
	275	0.670001147	0.6191599
	285	0.671366624	0.6203427
	325	0.669771592	0.6191255
	343	0.659192099	0.6090224
	360	0.656571262	0.6060595
	375	0.650316108	0.6011111
	445	0.593393238	0.5719513
	475	0.547726996	0.5477277
Low shot size setting	90	0.672972285	0.6124695
	145	0.672809622	0.6270955
	170	0.669223792	0.6228778
	195	0.668933217	0.6224893
	215	0.667953017	0.6198099
	235	0.664037217	0.6172277
	275	0.662142917	0.6153314
	285	0.659374749	0.6123502
	325	0.657487586	0.6111499
	343	0.654271382	0.6086336
	360	0.636188927	0.5909422
	375	0.625708532	0.5815222
	445	0.500464687	0.4779042
	475	0.473823163	0.4583211
High melt temperature	490	0.457995038	0.4428888
	90	0.630364074	0.6512945
	145	0.690318652	0.6464811
	170	0.687879622	0.6438853
	195	0.685119826	0.6393128
	215	0.684136001	0.6399226
	235	0.679103836	0.6349884
	275	0.679561568	0.6331024
	285	0.672891331	0.6284004
	325	0.67031	0.6252611
	343	0.66521	0.6222066
	360	0.65222	0.609613
	375	0.6294	0.5871462
	445	0.575210798	0.5533727
	460	0.539712115	0.5219994
	475	0.529919918	0.5144412
	490	0.527618877	0.5126511
Low melt temperature	505	0.515450591	0.5037386
	90	0.686706115	0.6402992
	145	0.679521206	0.638394
	170	0.684293066	0.630512
	195	0.689236851	0.6349889
	215	0.684609911	0.627686
	235	0.686483024	0.632174
	260	0.681287794	0.6269152
	275	0.677176117	0.622611
	285	0.672125212	0.617256
	315	0.668292035	0.6125155
	325	0.667941945	0.6117051
	343	0.665278909	0.6104099
	360	0.661518419	0.6076222
	375	0.65875516	0.6050851
	460	0.562243493	0.540819
	475	0.544277894	0.52646
	490	0.532056223	0.5165548
	525	0.511946383	0.496839